Journal of Earthquake Engineering

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/ueqe20

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JOURNAL OF EARTHQUAKE ENGINEERING
https://doi.org/10.1080/13632469.2021.1911884

Effect of Radiation Damping on the Fundamental Period of Linear Soil Profiles

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ABSTRACT

The site fundamental period plays a key role in the characterization of site conditions to construct design spectrum models. For layered soil profiles on bedrock, it is generally believed that radiation damping caused by energy leaking into the bedrock does not change the fundamental period. This study finds that radiation damping can significantly affect the fundamental period. The effect of radiation damping on the site fundamental period is systematically studied, and a simple method for the fundamental period of linear soil profiles considering radiation damping is proposed. The validity of the proposed method is demonstrated using many actual soil profiles.

ARTICLE HISTORY

Received 07 April 2020
Accepted 26 March 2021

KEYWORDS

The site fundamental period; radiation damping; layered soil profile; wave propagation theory; site condition

1. Introduction

The site fundamental period has been adopted as a main site classification parameter to construct design spectra not only in many studies (Cadet, Bard, and Rodriguez-Marek 2012; Pitilakis, Riga, and Anastasiadis 2013) but also in some codes and recommendations, e.g., the Japan Road Association (1980, 1990) and AIJ Recommendations for Loads on Buildings (2012, 2015).

The site fundamental period is often estimated by considering the site as a layered soil profile on bedrock. Since the fundamental period of a vibration system represents the longest period at which resonance between the system and periodic input motion occurs, the site fundamental period corresponds to the first peak of the soil profile's transfer function. The soil profile's transfer function is defined as the ratio of Fourier spectra between the ground surface and outcrop bedrock, i.e., $2A_S/2A_B$, where A_S and A_B are surface and bedrock incident wave amplitudes, respectively. For layered soil profiles on the assumed rigid bedrock, the peaks of the transfer function also occur at the modal natural periods; thus, the fundamental period is identical to the first modal natural period. For layered soil profiles on real elastic bedrock, energy can leak into the bedrock and cause radiation damping. In this case, exact modal analysis cannot be carried out (Zhao 1996, 1997); therefore, the modal natural periods do not theoretically exist. Nevertheless, current studies generally consider that the fundamental period still equals the first modal natural period of the soil profiles on the assumed rigid bedrock and that radiation damping caused by energy leaking to the bedrock does not affect the fundamental period (Sarma 1994; Vijayendra, Nayak, and Prasad 2015; Zhang and Zhao 2017, 2018; Zhao 1996, 1997). The fundamental period and first modal natural period are commonly used interchangeably; both of them are always simply estimated using the methods developed based on layered soil profiles on assumed rigid bedrock without considering radiation damping (Dobry, Oweis, and Urzua 1976; Hadjian 2002; Madera 1970).

If radiation damping does not affect the site fundamental period, the first peak of the soil profile's transfer function should occur at the same period regardless of whether radiation damping exists. However, during an analysis of an actual soil profile on elastic bedrock in Japan, it was found that the first peaks of the transfer functions with and without consideration of radiation damping occurred at

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different periods. Figure 1 shows the shear wave velocity profile of the soil profile. Figure 2 presents the transfer functions of the soil profile on the original elastic bedrock, i.e., considering radiation damping, and on the assumed rigid bedrock, i.e., without considering radiation damping. It is found that the fundamental period is shifted significantly by the radiation damping, which means that the fundamental period can be affected by radiation damping. The same phenomenon was also observed by Kokusho (2013, 2017) by analyzing seismic records in Japan and comparing them with theoretical transfer functions with and without radiation damping.

The objective of this paper is to investigate the effect of radiation damping on the site fundamental period. In Section 2, the effect of radiation damping on the fundamental period is explored using two simple soil models. In Section 3, based on the analyses in Section 2, a simple method for determining the site fundamental period considering radiation damping is developed. In Section 4, the validity of the proposed method is demonstrated using many actual soil profiles. Finally, the conclusions are presented in Section 5.

2. Effect of Radiation Damping on the Site Fundamental Period

To explore the effect of radiation damping on the site fundamental period, this section considers two simplest soil models and discusses the problem of basic definitions based on wave propagation theory.

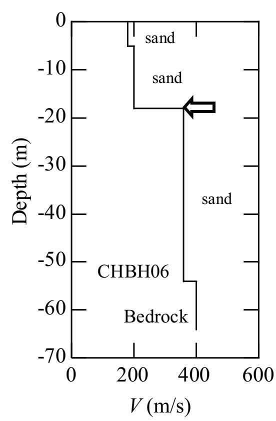

Figure 1. Shear wave velocity profile of the actual soil profile.

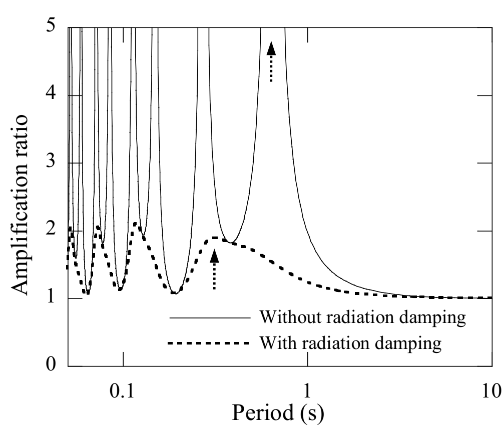

Figure 2. Transfer functions with and without consideration of radiation damping.

2.1. A Single-Layer Soil Profile on an Elastic Half-Space

First, a single-layer soil profile in an elastic half-space, as shown in Fig. 3, is considered. Then, the effect of radiation damping on the fundamental period of the soil model is explored in the time and frequency domains in the following two subsections, respectively.

2.1.1. Exploration in the Time Domain

To explore the effect of radiation damping on the fundamental period in the time domain, an incident sinusoidal seismic wave traveling along a perfectly vertical path approaching the soil–bedrock interface is considered Fig. 3. According to wave propagation theory, when an incident wave with displacement amplitude A_I, perpendicularly reaches an interface between two different media, part of its energy will be transmitted across the interface with displacement amplitude A_T. The remaining energy will be reflected back with displacement amplitude A_R, the displacement amplitudes of the transmitted and reflected waves can be obtained by

$$Tr = \frac{A_T}{A_I} = \frac{2}{1 + a_I} \tag{1}$$

$$Re = \frac{A_R}{A_I} = \frac{1 - a_I}{1 + a_I} \tag{2}$$

where Tr and Re are the transmission and reflection coefficients, respectively; and a_I is the impedance ratio of the media in which the transmitted and incident waves propagate.

The transmitted wave will propagate upward and reach the ground surface, as shown in Fig. 3. At this moment, the time t is considered as 0 s, and the displacement function is assumed to be

$$y_0(t) = A_0 sin(\omega t) \tag{3}$$

where A_0 and ω are the displacement amplitude and angular frequency of the wave, respectively. Eq. (2) implies that when the wave reaches the ground surface (i.e., $a_I = 0$), the reflected wave will have the same amplitude and polarity as the incident wave. Therefore, the wave reflected from the ground surface will propagate downward with amplitude A_0. Ignoring the energy loss caused by soil material damping, the wave will reach the soil–bedrock interface with amplitude A_0 after H/V seconds, where H and V are the thickness and shear wave velocity of the soil layer, respectively. Similarly, the downwardly propagating wave will be reflected back from the soil–bedrock interface by the same mechanism as described above. Eq. (2) implies that when the wave propagates from the soft-soil layer to the stiffer bedrock (i.e., $a_I > 1$), the amplitude of the reflected wave A_1 will be $|Re|A_0$, and the polarity of the reflected wave will be opposite to that of the incident wave. Because $|Re|$ is naturally less than 1, A_1 will be smaller than A_0, which means that the displacement amplitude will be reduced every time the seismic wave is reflected by the stiffer bedrock. Essentially, the amplitude reduction is due to the

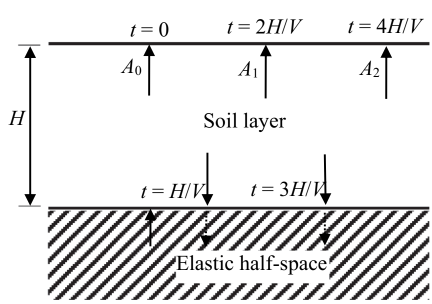

Figure 3. Single-layer soil profile on an elastic half-space.

energy loss at the soil–bedrock interface, which is known as radiation damping. Then, the wave reflected from the soil–bedrock interface will propagate upward and reach the ground surface after more H/V seconds. Compared with the initial incident seismic wave at the ground surface (t = 0 s), the seismic wave reflected back to the ground surface (t = 2 H/V) has reduced displacement amplitude, opposite polarity and reaches the ground surface 2 H/V seconds later. Therefore, the displacement of the seismic wave reflected back to the ground surface can be expressed as

$$y_1(t) = -A_1 sin(\omega(t - 2H/V)) \tag{4}$$

The seismic wave will continue to propagate and be reflected at the soil–bedrock interface. It can be deduced from the above discussion that when a seismic wave propagates one cycle from the ground surface to the soil–bedrock interface and back to the ground surface, the amplitude will be reduced to $|Re|$ times the previous amplitude, the polarity will become reverse, and 2 H/V seconds will have passed. Therefore, the displacement of a seismic wave reflected back to the ground surface after propagating k (k = 0, 1, 2 . . .) cycles can be expressed as

$$y_k(t) = (-1)^k A_k sin(\omega(t - 2kH/V)) \tag{5}$$

where A_k represents the displacement amplitude of the seismic wave reflected back to the ground surface after propagating k cycles. It can be expressed as

$$A_k = |Re|A_{k-1} \tag{6}$$

Eqs. (3)–(5) represent the displacement of the upwardly propagating waves at the ground surface. The real displacement at the ground surface equals the sum of the displacements of the upwardly and downwardly propagating waves. Eq. (2) implies that the upwardly propagating incident wave and downwardly propagating reflected wave at the ground surface have same displacement and polarity. Thus, the displacement at the ground surface induced by the seismic wave propagating k cycles equals $2y_k(t)$. For an infinite incident seismic wave, all of the seismic waves propagating k ($k = 0 - \infty$) cycles will exist at the ground surface at the same time; therefore, the displacement at the ground surface can be expressed as

$$y_S(t) = \sum_{k=0}^{\infty} 2y_k(t) \tag{7}$$

Based on Eq. (7), the resonance periods of the soil profile can be obtained. Resonance periods are the periods at which resonance between the soil profile and incident seismic waves occurs. More precisely, the resonance periods equal the periods of the incident waves that cause the displacement amplitude $2A_S$ at the ground surface to become maximum, i.e.,$2A_S = |y_S(t)|_{\max}$. Eqs. (5) and (7) indicate that A_S is a function of ω, when the maximum displacement $|y_k(t)|_{\max}$of each seismic wave reflected back to the ground surface occurs at the same time, $2A_S$ becomes maximum

$$(2A_S(\omega))_{\max} = \sum_{k=0}^{\infty} 2A_k \tag{8}$$

To make the maximum displacement of each seismic wave occur at the same time, it requires that phase of the seismic wave shifts $(2n - 1)\pi$ every time it propagates one cycle from the ground surface to the soil–bedrock interface and back to the ground surface,

$$\omega \times 2H/V = (2n - 1)\pi \tag{9}$$

Here, n is a natural number (n = 1, 2, 3 . . .). This is because the $(2n - 1)\pi$ phase shift can just make the changed polarity of the seismic wave caused by the reflection at the soil–rock interface return to the origin. Substituting Eq. (9) into Eq. (5) results in

$$y_k(t) = A_k sin(\omega t) \tag{10}$$

Eq. (10) can confirm that when ω satisfies Eq. (9), the maximum displacement of each seismic wave reflected back to the ground surface occurs at the same time and the displacement amplitude at the ground surface reaches the maximum. Therefore, according to Eq. (9), resonance periods of the soil profile T_R^n can be expressed as

$$T_R^n = T_R^1/(2n-1) \tag{11}$$

where T_R^1 is the first resonance period, i.e., the site fundamental period

$$T_R^1 = \frac{4H}{V} \tag{12}$$

According to Eq. (12), the effect of radiation damping on the fundamental period can be clarified. Eq. (12) indicates that the fundamental period is a function of H and V of the soil layer and is independent of the properties of the half-space. This means that despite whether the half-space is elastic or rigid (i.e., there is radiation damping or not), the fundamental period does not change. Therefore, the fundamental period is not affected by radiation damping. It should be noted that Eq. (12) is also well known as the equation for the first modal natural period of a single-layer soil profile on a rigid half-space. Therefore, for a single-layer soil profile on an elastic half-space, the fundamental period is not affected by radiation damping and is identical to the first modal natural period of the soil profile on an assumed rigid half-space.

2.1.2. Exploration in the Frequency Domain

The effect of radiation damping on the fundamental period can be explored from another perspective in the frequency domain. Given that the fundamental period corresponds to the first peak of the transfer function, the transfer function $H_{T1}(\omega)$ of the simple soil model is obtained as

$$H_{T1}(\omega) = \frac{1}{\cos\frac{\omega T_R^1}{4\sqrt{1+2ih}} + ia\sin\frac{\omega T_R^1}{4\sqrt{1+2ih}}} \tag{13}$$

where h is the soil damping ratio, i is the complex number ($i^2 = -1$), a is the impedance ratio, which is defined as

$$a = \frac{\rho V}{\rho_B V_B} \tag{14}$$

Here, ρ is the density of the soil layer, and ρ_B and V_B are the density and shear wave velocity of the elastic half-space, respectively.

Then, the fundamental period of the soil profile can be obtained according to Eq. (13). The peak of the site transfer function occurs when the denominator of Eq. (13) becomes minimum. The denominator of Eq. (13) is a complex-valued function; for undamped soil (i.e., $h = 0$), the real part, $\cos\frac{\omega T_R^1}{4}$, defined as X, and the imaginary part, $a\sin\frac{\omega T_R^1}{4}$, defined as Y, satisfy an elliptic equation:

$$X^2 + \frac{Y^2}{a^2} = 1 \tag{15}$$

The semi-major and semi-minor axes of the ellipse are 1 and a ($a < 1$), respectively. The distance from a point on the ellipse to point (0, 0) (i.e.,$\sqrt{X^2 + Y^2}$) is the absolute value of the denominator of Eq. (13). This absolute value becomes minimum and equals the semi-minor axis of the ellipse a when the real and imaginary parts satisfy $X = 0$ and $|Y| = a$, respectively. This requires ω to satisfy

$$\omega = \frac{2(2n-1)\pi}{T_R^1} \tag{16}$$

In other words, when the period satisfies

$$T_R^n = \frac{2\pi}{\omega} = \frac{T_R^1}{2n-1} \tag{17}$$

the denominator of Eq. (13) becomes minimum and the peaks of the transfer function will occur. Therefore, Eq. (17) represents the resonance periods of the single-layer soil profile, and the fundamental period equals T_R^1. It should be noted that Eq. (17), derived in the frequency domain, is exactly the same as Eq. (11), derived in the time domain. Therefore, the conclusion regarding the effect of radiation damping on the fundamental period obtained in the frequency and time domains should be consistent.

In the above discussion, material damping is ignored, which exists in real soil sites. In fact, even soil damping h is considered, the conclusion derived above is still valid. The site fundamental period considering soil damping can be obtained as $T_R^1/\sqrt{1+2ih}$ using Eq. (13) by imitating the analysis in the frequency domain, which is also independent of the properties of the half-space.

To confirm this point, several example calculations are further conducted. The fundamental period T_f of the soil model used is set as 0.3 s. Two soil damping ratios 0 and 0.05 are considered. Three impedance ratios 0, 0.1, and 0.5 are considered to represent three different levels of radiation damping. Then, the site transfer functions are calculated, and the obtained results are shown Fig. 4. It is found that the peaks of the transfer function occur at the same periods regardless of radiation damping. The results support the idea that the fundamental period is not affected by radiation damping for the single-layer soil profile.

In the above calculation, the difference in radiation damping is reflected in terms of the impedance ratio a. This is because Eq. (1) indicates that the displacement amplitude of a wave transmitted into the half-space is determined by a; the displacement amplitude of the transmitted wave is in turn proportional to the radiation damping caused by energy leaking into the half-space; therefore, different impedance ratios correspond to different radiation damping.

2.2. A Two-Layer Soil Profile on an Elastic Half-Space

To further explore the effect of radiation damping on the site fundamental period, a two-layer soil profile in an elastic half-space is considered, as shown in Fig. 5. To explore whether radiation damping can change the fundamental period of the two-layer soil profile, two extreme cases are considered. Case 1: the rigidity of the half-space is infinite, which prevents energy from leaking into the half-space; therefore, radiation damping does not exist. Case 2: the half-space is elastic with the same rigidity as that of the second soil layer, which allows energy to leak into the half-space; therefore, radiation damping does exist. In the two cases, soil material damping is ignored, and the densities of the soil layers and the half-space are considered the same.

For Case 1, because the half-space is rigid, resonance periods are identical to the modal natural periods. The equation for the first modal natural period of a two-layer soil profile on a rigid half-space has been derived by Madera (1970)

$$\tan\frac{\pi T_1}{2T_f}\tan\frac{\pi T_2}{2T_f} = \frac{\rho_2 H_2 T_1}{\rho_1 H_1 T_2} \tag{18}$$

Here, ρ_1 and H_1 are the density and thickness of the first soil layer, respectively; ρ_2 and H_2 are the density and thickness of the second soil layer, respectively; and T_1 and T_2 are defined, respectively, as

$$T_1 = \frac{4H_1}{V_1} \tag{19}$$

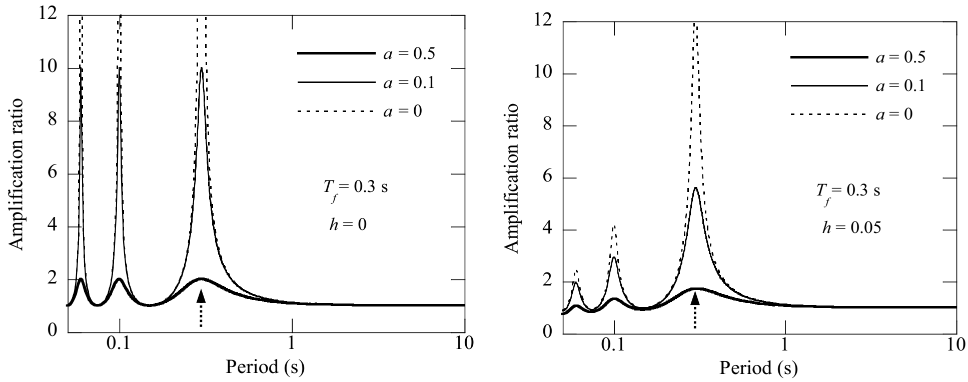

Figure 4. Transfer functions of the single-layer soil profile with various radiation damping.

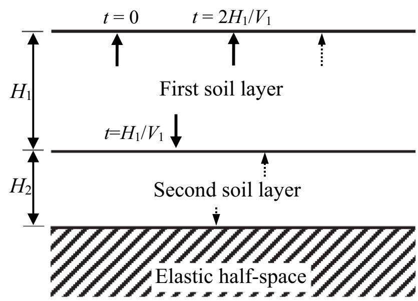

Figure 5. Two-layer soil profile on an elastic half-space.

$$T_2 = \frac{4H_2}{V_2} \tag{20}$$

where V_1 and V_2 are the shear wave velocities of the first and second soil layers, respectively. Therefore, the fundamental period T_f for Case 1 without radiation damping can be obtained from Eq. (18).

For Case 2, since the rigidity of the half-space is the same as that of the second soil layer, the two-layer soil profile on the elastic half-space can be considered as a one-layer soil profile on a new elastic half-space, which is composed of the original half-space and second soil layer. Therefore, the resonance periods for Case 2 with radiation damping equal those of the first soil layer. According to the discussions in Section 2.1, the resonance periods for Case 2 can be expressed as

$$T_R^n = T_1/(2n-1) \tag{21}$$

Therefore, the fundamental period for Case 2 with radiation damping equals T_1, which is the fundamental period of the first soil layer in an elastic half-space according to Eq. (19). The two cases are different only in radiation damping owing to the difference in rigidity of the half-space, but the fundamental period for Case 1, expressed by Eq. (18), is clearly different from that for Case 2, expressed by Eq. (19). Therefore, for a two-layer soil profile in an elastic half-space, the fundamental period can be affected by radiation damping.

Furthermore, the transfer functions of several example two-layer soil profiles on elastic and rigid half-spaces (i.e., with and without radiation damping) are calculated, and the results are shown in Fig. 6. The parameters of the example soil profile are also shown in the figure. ρ_B and V_B represent the density and shear wave velocity of the half-space, respectively. It is found that the fundamental periods with and without radiation damping can be clearly different, which further proves that radiation damping can affect the fundamental period of the two-layer soil profile. It should be noted that the sites in Fig. 6 are designed to explain the effect of radiation damping on the fundamental period, though they may be rarely encountered in reality.

To systematically study the effect of radiation damping on the fundamental period, the equation for the transfer function $H_{T2}(\omega)$ of the two-layer soil profile is obtained as (Zhang and Zhao 2018)

$$H_{T2}(\omega) = \frac{1}{(\cos C_1 \cos C_2 - a_1 \sin C_1 \sin C_2) + i(a_1 a_2 \sin C_1 \cos C_2 + a_2 \cos C_1 \sin C_2)} \tag{22}$$

where,

$$C_1 = \frac{\omega T_1}{4\sqrt{1 + 2ih_1}} \tag{23}$$

$$C_2 = \frac{\omega T_2}{4\sqrt{1 + 2ih_2}} \tag{24}$$

and, h_1 and h_2 are the damping ratios of the first and second soil layers, respectively. a_1 is the impedance ratio of the first soil layer with respect to the second soil layer

$$a_1 = \frac{\rho_1 V_1}{\rho_2 V_2} \tag{25}$$

a_2 is the impedance ratio of the second soil layer with respect to the half-space

$$a_2 = \frac{\rho_2 V_2}{\rho_B V_B} \tag{26}$$

It can be known from Eq. (22) that for a two-layer soil profile in an elastic half-space, six parameters including a_1, a_2, T_1, T_2, h_1, and h_2 control the transfer function and hence the fundamental period. To systematically study the effect of radiation damping on the fundamental period, two-layer soil profiles with a wide range of values for these parameters are considered. Precisely, a_1 varies from 0.2 to 1.0, a_2 varies from 0 to 0.95, T_2/T_1 varies from 0.05 to 5 and h_1 and h_2 vary from 0 to 0.05. Then, the fundamental period T_f of these soil profiles considering radiation damping is obtained from the transfer function calculated by Eq. (22), and the obtained results are compared with those obtained by Eq. (18) ignoring radiation damping by assuming the half-space as rigid. The fundamental period T_f considering radiation damping is determined according to the first peak of the transfer function calculated by Eq. (22), as illustrated in Fig. 6 by arrows for different conditions of H_1 = 6, 4, and 2.6 m. The peak of the transfer function is searched numerically using MATLAB according to its definition (i.e., $H_{T2}(T_f) > H_{T2}(T_f + \Delta T) \cap H_{T2}(T_f) > H_{T2}(T_f - \Delta T)$) then confirmed by eyes. Here, the period interval ΔT is adopted to be 0.001 s. Some representative results are shown in Figs. 7 and 8. In these

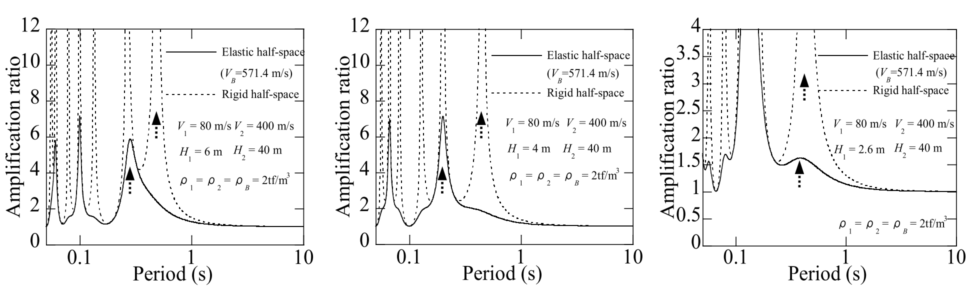

Figure 6. Transfer functions of a two-layer soil profile with and without radiation damping.

figures, the solid line represents the normalized results of the fundamental period T_f/T_1 considering radiation damping, and the dotted line represents the corresponding results ignoring radiation damping.

Figures 7 and 8 indicate that when the impedance ratio a_2 is small, the fundamental periods ignoring radiation damping are almost identical to those considering radiation damping. As a_2 increases, the error caused by ignoring radiation damping increases. In addition, the error of the fundamental period caused by ignoring radiation damping is found to be affected by the impedance ratio a_1, it increases as a_1 decreases.

In addition, Figs. 7c,d and 8c,d indicate that when a_2 increases to a certain extent and a_1 simultaneously decreases to a certain extent, the curves of the fundamental period considering radiation damping become stepped. The stepped curves for $a_2 = 0.7$ and $a_1 = 0.2$, as representatives are represented by the bold line in Figs. 7 and 8d. It is found that when T_2/T_1 is smaller than a certain value, the value of T_f/T_1 suddenly decreases almost to 1. The point at which the value of T_f/T_1 suddenly decreases is called the turning point, as shown in Figs. 7d and 8d. It is at this point that the error of the fundamental period caused by ignoring radiation damping is most significant. It is noted that for nearly all the cases, the fundamental period considering radiation damping is shorter than that ignoring radiation damping, i.e., radiation damping can shorten the fundamental period. In addition, the same characteristics shown in Figs. 7 and 8 indicate that soil damping does not affect the trend of the fundamental period with radiation damping, although the fundamental period varies slightly with soil damping.

To explain why T_f/T_1 may approach 1, i.e., the fundamental period of the two-layer soil profile approaches that of the first soil layer, when a_2 is large and a_1 is small; a sinusoidal seismic wave traveling approaching the ground surface, as shown in Fig. 5 is considered again. According to Eqs. (1) and (2), when a_1 is small, most of the energy of the wave in the first soil layer will be reflected back when it reaches the interface of the two soil layers, and only a little energy can transmit into the second soil layer. Furthermore, when a_2 is simultaneously large, most of the energy transmitted into the second soil layer will leak into the half-space, and less energy can be reflected back. Therefore, the seismic waves reflected back to the ground surface are mainly from the interface of the two soil layers. Consequently, the displacement at the ground surface of the two-layer soil profile will be nearly equal to that of the first soil layer in an elastic half-space with the properties being the same as those of the second soil layer. According to the discussions in Section 2.1.1, the resonance periods are determined by the displacement at the ground surface. In addition, the equation for the resonance periods of the first soil layer on an elastic half-space can be easily derived as $T = T_1/(2n-1)$; hence, the fundamental period equals T_1. Therefore, the fundamental period T_f of the two-layer soil profile approaches that of the first soil layer on an elastic half-space T_1, i.e., T_f/T_1 approaches 1, when a_2 is large and a_1 is small.

The discontinuous change of the fundamental period can be explained by referring to Fig. 6. The three figures from left to right correspond to cases with $a_1 = 0.2$, $a_2 = 0.7$ on the far left ($T_2/T_1 = 1.3$), left ($T_2/T_1 = 2$), and right ($T_2/T_1 = 3.1$) of the turning point, respectively. It is found that, when T_2/T_1 is at the right of the turning point, the first peaks of transfer functions with and without consideration of radiation damping occur at similar periods. However, when the value of T_2/T_1 decreases to the turning point, the original first peak of the transfer function considering radiation damping disappears and the second peak replaces it. Since such a change is discontinuous, the fundamental period changes suddenly and discontinuously.

The discontinuous change of the fundamental period can be further explained from the wave propagation perspective. According to Eqs. (1) and (2), when a_2 is small and a_1 is large, most energy will reach the soil–bedrock interface and be reflected back to the ground surface. The resonance caused by seismic waves traveling complete circles from the ground surface to the soil–bedrock interface and back to the ground surface will be significant. The resonance period corresponds to T_f estimated by Eq. (18). However, when a_1 is small and a_2 is large, the resonance caused by seismic

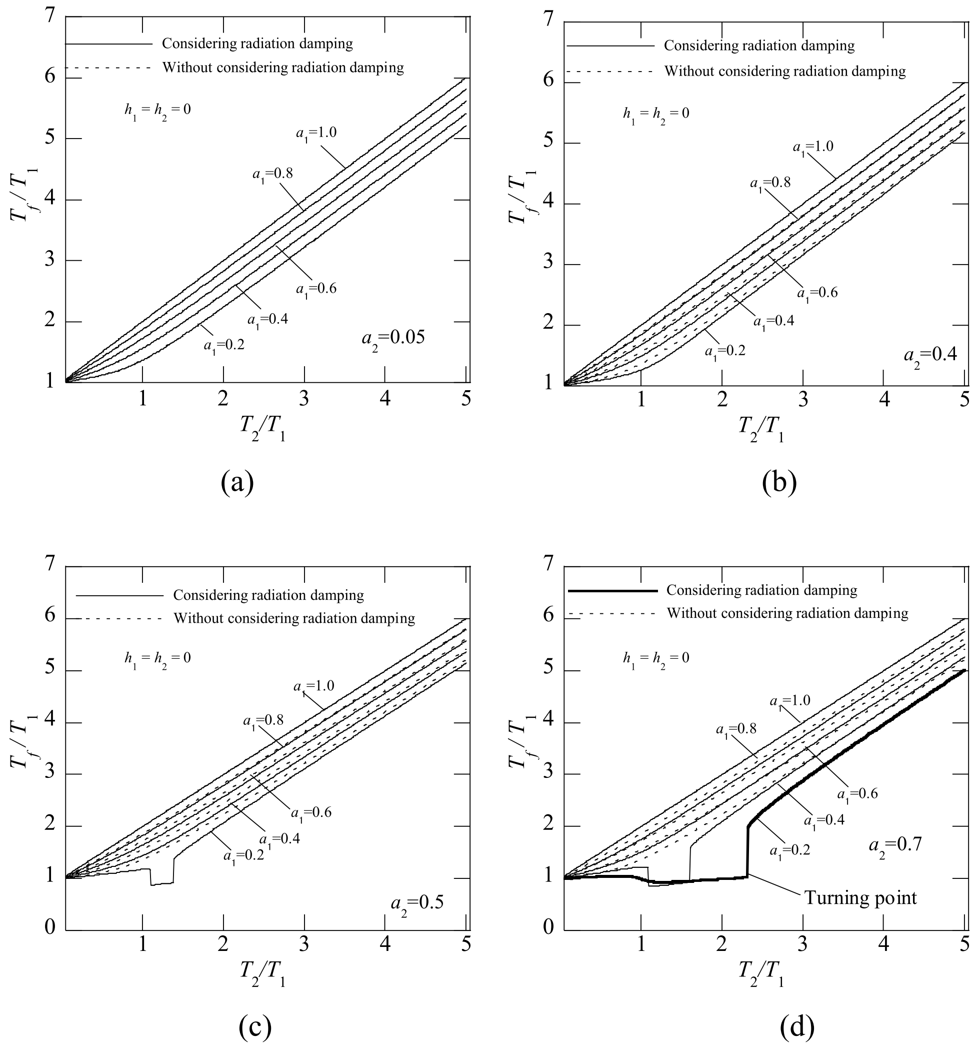

Figure 7. Comparison of the undamped fundamental periods considering and ignoring radiation damping, when (a) a_2 = 0.05; (b) a_2 = 0.4; (c) a_2 = 0.5; (d) a_2 = 0.7.

waves reflected from the interface of the two soil layers back to the ground surface will be more significant. The resonance period corresponds to T_1 estimated by Eq. (19). When the two resonance periods are far away (T_2/T_1 is large), the two corresponding peaks exist in the transfer function, the fundamental period is still T_f. However, when the two resonance periods are near enough (T_2/T_1 approaches the turning point), the larger peak at T_1 replaces the smaller one at T_f, the fundamental period changes suddenly.

The above conclusion can be easily extended to a general multilayer soil profile on bedrock. That is when the impedance ratio of the lowest soil layer with respect to the bedrock is large, and there are two adjacent soil layers with a large contrast (or their impedance ratio is very small), the fundamental period of the total soil profile may be shortened to that of the soil profile upon the interface of the two soil layers by radiation damping. This can be supported by the actual soil profile shown in Figs. 1 and 2. Therefore, it can be concluded that for a single-layer soil profile on bedrock, radiation damping does not affect the fundamental period, but for a general multilayer soil profile on bedrock, the radiation damping can affect the fundamental period.

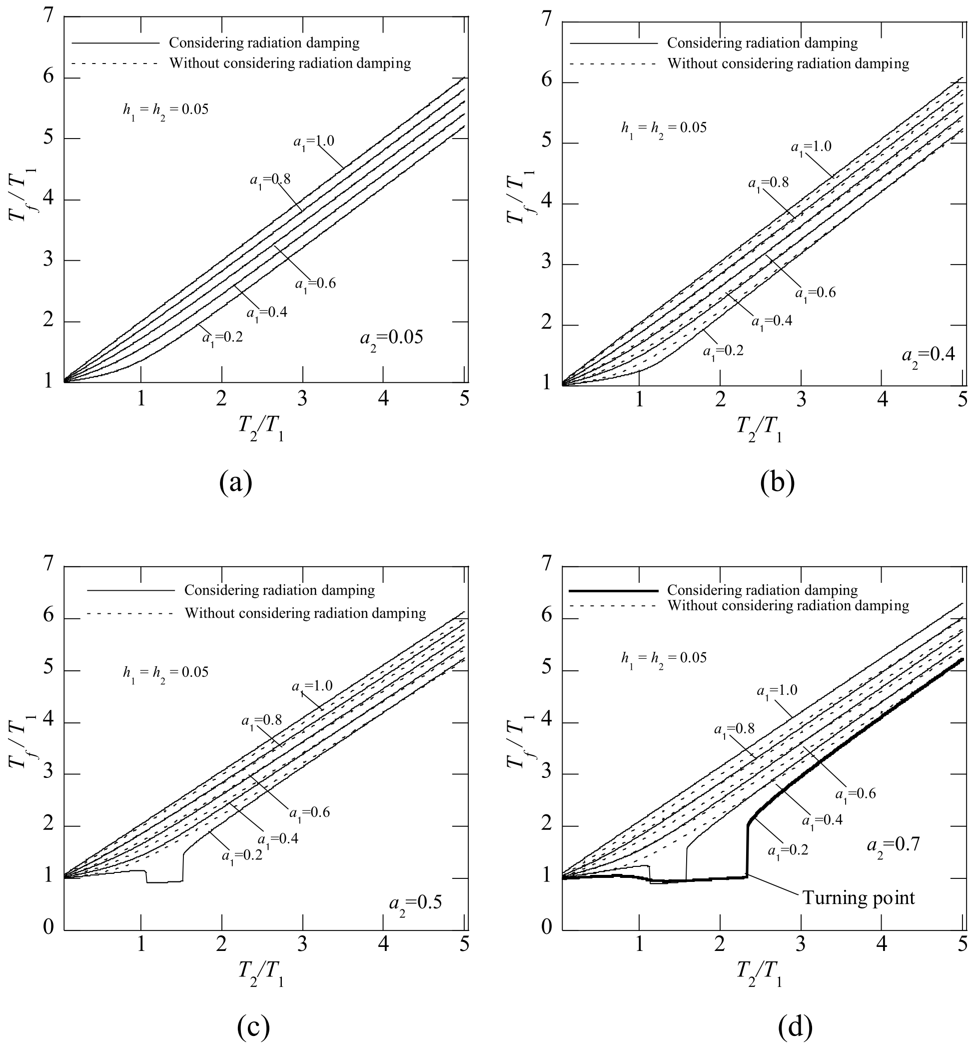

Figure 8. Comparison of the damped fundamental periods considering and ignoring radiation damping, when (a) a_2 = 0.05; (b) a_2 = 0.4; (c) a_2 = 0.5; (d) a_2 = 0.7.

3. Simple Method for the Site Fundamental Period considering Radiation Damping

The development of simple methods for determining the site fundamental period has been the focus of numerous studies from a very early time (Dobry, Oweis, and Urzua 1976; Hadjian 2002; Madera 1970). However, almost all the simple methods were developed based on layered soil profiles of the assumed rigid bedrock ignoring radiation damping, which may significantly affect the fundamental period as discussed above. In this section, a simple method for determining the site fundamental period considering radiation damping is developed.

Someone may doubt that when radiation damping is critical, the amplification reduces and the resonance at the fundamental period is unimportant. In fact, when radiation damping is sufficiently critical to shift the fundamental period, the amplification ratio at the fundamental period may still be important as shown in Fig. 6. The amplification ratio at the fundamental period of the actual site shown in Fig. 1 is about 2, and those observed by Kokusho (2013, 2017) from seismic records in Japan can be as large as about 6. Therefore, the estimation of the site fundamental period considering radiation damping is important.

3.1. The Fundamental Period for a Two-Layer Soil Profile

To develop a simple method for the fundamental period of multilayer soil profiles considering radiation damping, a method for a two-layer soil profile is first discussed. The equation for the fundamental period of a two-layer soil profile on a rigid half-space ignoring radiation damping was derived by Madera (1970) and expressed as Eq. (18). Simplified equations for Eq. (18), which can directly give results of the fundamental period, were developed by Hadjian (2002)

$$\frac{T_f}{T_1} = 1 + \frac{H_1}{H_2}\left(\frac{T_2}{T_1}\right)^2 \quad \text{for } T_2/T_1 \leq 1 \tag{27}$$

$$\frac{T_f}{T_1} = \sqrt{\frac{\pi^2}{8}\left[0.75 + \left(\frac{T_2}{T_1}\right)^2\left(1 + 2\frac{H_1}{H_2}\right)\right]} \quad \text{for } H_1/H_2 > 1 \tag{28}$$

$$\frac{T_f}{T_1} = \left[1 + \beta\left(\frac{T_2}{T_1}\right)^n\left(1 + \frac{H_1}{H_2}\right)^n\right]^{\frac{1}{n}} \quad \text{for } H_1/H_2 \leq 1 \tag{29}$$

Here,

$$n = 4 - 1.8\frac{H_1}{H_2} \tag{29–1}$$

$$\beta = 1 - 0.2\left(\frac{H_1}{H_2}\right)^2 \tag{29–2}$$

However, at present there is still no equation for the fundamental period of a two-layer soil profile in an elastic half-space considering radiation damping. It is found in Section 2.2 that the fundamental periods considering and ignoring radiation damping are significantly different when T_2/T_1 is smaller than the turning point but not that different for other cases. In addition, when T_2/T_1 is smaller than the turning point, the fundamental period of the two-layer soil profile considering radiation damping is nearly equal to that of the first soil layer. The idea herein is that by using the characteristics of the fundamental period considering radiation damping in conjunction with the simple equations for the fundamental period ignoring radiation damping; a simple method for the fundamental period of a two-layer soil profile considering radiation damping can be developed. Precisely, when T_2/T_1 is smaller than the turning point, the fundamental period of the two-layer soil profile considering radiation damping is equal to that of the first soil layer; in other cases, the fundamental period considering radiation damping is approximately equal to that ignoring radiation damping, as estimated by Eqs. (27)–(29).

Clearly, to develop the method for the fundamental period considering radiation damping is actually to find the turning point. It has been found that when the impedance ratios a_1 and a_2, respectively, decrease and increase to a certain extent, the curves of the fundamental period considering radiation damping become stepped. Furthermore, when the value of T_2/T_1 decreases to a certain extent, the turning point of the stepped curve occurs. According to the results obtained in Section 2.2, the critical values of a_1 and a_2 when the stepped curve occurs are plotted in (Fig. 9). Then, using these values, a simple equation is regressed as

$$a_1 = e^{3a_2}/20 \tag{30}$$

Thus, Eq. (30) can be used to judge when the stepped curve occurs. When$a_1 \leq e^{3a_2}/20$, the point (a_2, a_1) locates to the right side of the solid line, and the curve of the fundamental period considering radiation damping becomes stepped. Otherwise, the curves are smoothed. Furthermore, according to the results obtained in Section 2.2, the values of the turning point are plotted in (Fig. 10). Similarly, using these results, a simple equation to determine the turning point is regressed as

$$Tp = c - m{a_1}^n \tag{31}$$

where:

$$m = 5.71 \times 10^{-3} a_2^{-17.39} + 5.52 \tag{31–1}$$

$$n = 7.39 \times 10^{-4} \times a_2^{-15.26} + 2.44 \tag{31–2}$$

$$c = 4.84 \times a_2^{4.36} + 1.32 \tag{31–3}$$

Thus, using Eqs. (30) and (31), the turning point can be determined. It is found from (Figs. 9 and 10) that both of the obtained equations have very good accuracy. It should be noted that since the paper is limited to linear analysis, soil nonlinear behavior is not considered and a soil damping ratio of 2.5% typically used for linear analysis is adopted in the derivation of Eqs. (30) and (31).

Generally speaking, the proposed method calculates the fundamental period of a two-layer soil profile considering radiation damping as follows. When the parameters of the estimated two-layer soil profile satisfy

$$\begin{cases} a_1 \leq e^{3a_2}/20 \\ T_2/T_1 \leq Tp \end{cases} \tag{32}$$

the fundamental period is equal to that of the first soil layer, i.e., $T_f = T_1$. Otherwise, the fundamental period equals that ignoring radiation damping estimated by Eqs. (27)–(29).

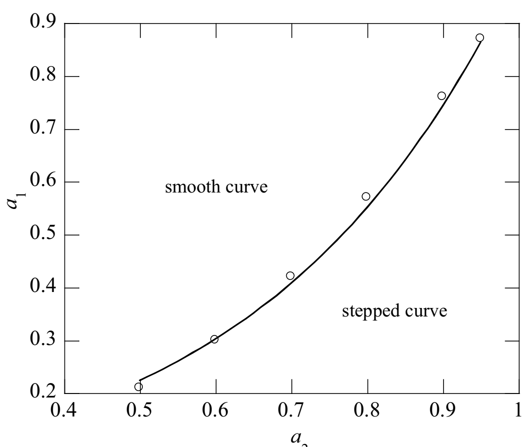

Figure 9. Critical values of impedance ratios a_1 and a_2 when the stepped curve occurs.

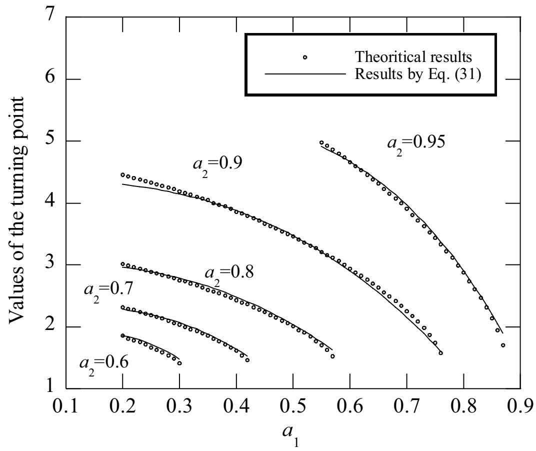

Figure 10. Values of the turning points for the stepped curves.

3.2. The Fundamental Period for Multilayer Soil Profiles

For multilayer soil profiles, the fundamental period considering radiation damping can be estimated successively using the equations for the two-layer soil profile in Section 3.1, based essentially on the idea of Madera (1970). As illustrated by (Fig. 11), the procedure starts by replacing the top two layers of the multilayer soil profile by an equivalent single layer. The equivalent single layer and the third layer of the multilayer soil profile are then treated as a new second two-layer system and are in turn replaced by an equivalent single layer. By the successive application of this procedure to the bedrock of the soil profile, the multiple soil layers are finally replaced by an equivalent single layer. Then, the fundamental period can be easily obtained. At each step of the replacement, the equations for the two-layer soil profile are used to make the equivalent single layer have the same fundamental period as the top two layers. Specifically, the developed procedure involves the following steps:

(1) For a multilayer soil profile on bedrock (Fig. 11a), the top two soil layers are assumed to overlie the bedrock and are replaced by an equivalent single layer. The thickness of the equivalent single-layer H_{eq1} equals the sum of those of the top two layers, i.e., $H_{eq1} = H_1 + H_2$. To make the equivalent single layer have the same fundamental period as the top two layers, the shear wave velocity of the equivalent single-layer V_{eq1} equals $4H_{eq_1}/T_f$. Here, T_f is the fundamental period of the top two layers considering radiation damping, which is estimated by the method proposed in Section 3.1. Then, a new multilayer soil profile (Fig. 11b) is formed.
(2) For the new multilayer soil profile, the top two layers are again assumed to overlie the bedrock and are replaced by another equivalent single layer. Similarly, the thickness of the equivalent single-layer H_{eq2} equals the sum of those of the top two layers, i.e., $H_{eq2} = H_{eq1} + H_2$, and the shear wave velocity of the equivalent single-layer V_{eq2} equals $4H_{eq_2}/T_f$. Here, T_f is the fundamental period of the new two layers considering radiation damping, which is also estimated by the method developed in Section 3.1.
(3) By successively applying the procedure until the last soil layer is considered, a final equivalent single layer is obtained, as shown in (Fig. 11d). Then, the fundamental period can be obtained quite easily.

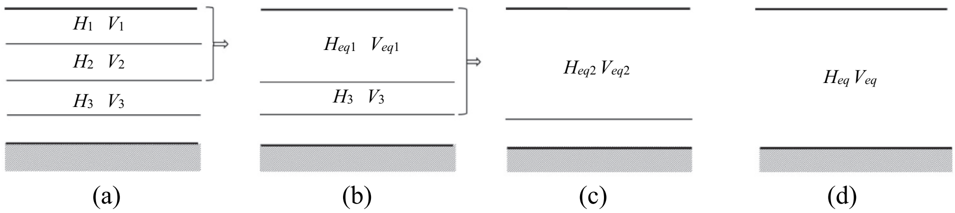

Figure 11. Concept of replacing multiple soil layers on bedrock by an equivalent single layer.

3.3. Application of the Proposed Method

The proposed method can be easily implemented on a spreadsheet, and the complexity is similar to that of the Hadjian (2002) method. To make the calculation easier, it is better to judge if radiation damping can significantly affect the results based on Eq. (32), before applying the proposed method to calculate the site fundamental period. If radiation damping can significantly affect the site fundamental period, the proposed method should be used; otherwise, traditional methods can be applied.

It is noted that Eq. (32) is developed for two-layer soil profiles. For general multilayer soil profiles, the multiple soil layers can be replaced with equivalent two layers based on the method by Zhang and Zhao (2017). Specifically, the interface between the equivalent two layers is located between two adjacent soil layers whose impedance contrast is largest among all soil layers. And the shear wave velocities and densities of the equivalent two layers are obtained by simply weighted averaging those of soil layers above and below the interface, respectively. If the parameters of the estimated soil profile satisfy Eq. (32), the effect of radiation damping is significant; otherwise, the effect is minor and can be neglected.

4. Verification of the Proposed Method

To investigate the accuracy of the developed method, various sites are selected from Strong-motion Seismograph Networks (K-NET, KiK-net) of Japan. The shear wave velocity profiles of the selected sites are presented in Figs. 12 and 13. Here, the soil profiles above the engineering bedrock defined in Japanese seismic code (Miura 2001) are considered. The unit weights are determined empirically according to Sakai et al. (2003) as 15.68 kN/m^3 for clay, 18.62 kN/m^3 for sand, 19.60 kN/m^3 for bedrock with shear wave velocity in the range of 400–800 m/s and 21.56 kN/m^3 for bedrock with shear wave velocity greater than 800 m/s. The soil damping is set as 2.5% that is typically adopted for linear analysis. The fundamental period of the selected soil profiles is calculated by the SHAKE program (Idriss and Sun 1992), and the results vary widely from 0.06 s to 1.67 s. It should be noted that, since the proposed method is limited to linear systems, soil nonlinear behavior is not considered in the analysis.

Before calculating the fundamental period, whether radiation damping has a significant effect on the fundamental period is judged based on the approach presented in Section 3.3. The soil profile shown in Fig. 1 is also estimated. The station code of this site is CHBH06. It is found that the effect of radiation damping on the fundamental period is minor for the four sites shown in Fig. 12, and significant for the four sites in Fig. 13 and the one in Fig. 1. The reason can be explained based on the conclusion in Section 2.2. For the soil profiles in Figs. 1 and 13, since the impedance ratio of the lowest soil layer with respect to the bedrock is large and there are two adjacent soil layers with a large contrast (shown by arrows), the fundamental period of the total soil profile can be shortened significantly by radiation damping. Instead, for the soil profiles in Fig. 12, since the impedance ratio of the lowest soil layer with respect to the bedrock is relatively small and contrasts between soil layers are not that large, the fundamental period is affected little by radiation damping.

Then, the fundamental period of these soil profiles is calculated using the proposed method considering radiation damping, and the obtained results are then compared with those from

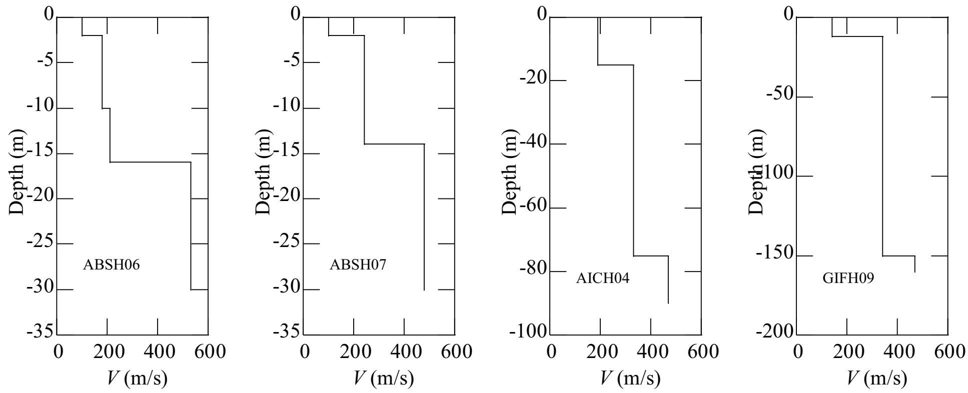

Figure 12. Shear wave velocity profiles of the selected sites with the fundamental period not being significantly affected by radiation damping.

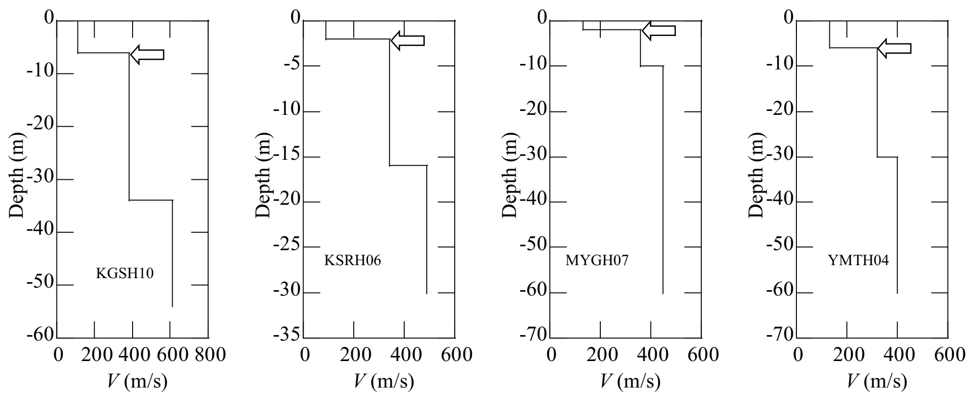

Figure 13. Shear wave velocity profiles of the selected sites with the fundamental period being significantly affected by radiation damping.

SHAKE in Table 1. It is found that all the results by the proposed method agree remarkably well with those from SHAKE. In addition, the fundamental period of these soil profiles is calculated using the method of Hadjian (2002), which is the most accurate simple method for the fundamental period ignoring radiation damping at present. It can be seen that, for the four sites in Fig. 12, the results by the Hadjian method agree well with those from SHAKE. However, for the four sites in Fig. 13 and the one in Fig. 1, the results are dramatically different from those of SHAKE, which is caused by ignoring radiation damping. These results are consistent with the above judgment, which proves the validity of the approach presented in Section 3.3.

The developed method is further verified by investigating observed transfer functions at these sites during real earthquakes. Transfer functions from vertical array records in EW and NS directions are shown in Fig. 14. The seismic records with medium peak ground accelerations (around 80 gal) were adopted to ensure an adequate signal-to-noise ratio and avoid soil non-linear effects. Since these transfer functions correspond to spectral ratios with respect to the inner bedrock, radiation damping is not involved. The transfer functions considering radiation damping (with respect to outcrop bedrock) are calculated according to Kokusho (2013, 2017) using the SHAKE program. Since seismometers were installed very deep in these sites instead of on the engineering bedrock, the soil profiles above the installed seismometers are considered

Table 1. Results of the fundamental period calculated by each method.

Site Code	SHAKE (s)	Proposed method (s)	Hadjian method (s)
ABSH06	0.32	0.34	0.34
ABSH07	0.23	0.23	0.23
AICH04	0.89	0.91	0.91
GIFH09	1.79	1.76	1.76
KGSH10	0.20	0.22	0.37
KSRH06	0.09	0.09	0.19
MYGH07	0.06	0.06	0.11
YMTH04	0.18	0.18	0.38
CHBH06	0.33	0.36	0.63

here. Figure 14 shows the results of four representative sites. Figure 14a is a representation for which the effect of radiation damping on the site fundamental period is significant (KGSH10 and ABSH06); Fig. 14b is a representation for which the effect of radiation damping is small (CHBH06 and AICH04); Fig. 14c is a representation for which the transfer function considering radiation damping has no very clear peaks (KSRH06 and MYGH07); Fig. 14d is a representation for which the fundamental period without considering radiation damping obtained by SHAKE analyses is not consistent with that from observations (YMTH04, ABSH07, and CIFH09). Results for the fundamental period calculated by the proposed method are represented by arrows. It should be noted that since the depths of soil profiles in Fig. 14 are different from those in Figs. 1, 12, and 13, their results are different, though they share the same station codes. It can be noted that for most cases, fundamental periods without considering radiation damping obtained by SHAKE analyses are consistent with those from observations. Although results from SHAKE analyses may disagree with those from observations (YMTH04, ABSH07, and CIFH09), results by the proposed method agree very well with those from SHAKE analyses for nearly all these cases. The inconsistency between results from SHAKE analyses (or the proposed method) with those from observations may be because three-dimensional properties of these sites cannot be adequately captured by the one-dimensional approximation applied in SHAKE analyses.

5. Conclusions

For layered soil profiles on bedrock, it is generally considered that radiation damping caused by energy leaking into the bedrock does not change the fundamental period. However, this study found that in some cases, radiation damping can significantly shorten the fundamental period. The effect of radiation damping on the fundamental period is systematically studied using two simple soil models, and a simple method for estimating the fundamental period of linear soil profiles considering radiation damping is developed. The validity of the proposed method is demonstrated using many actual soil profiles. It is found that,

(1) for a single-layer soil profile in an elastic half-space, the fundamental period is not affected by radiation damping, and it is identical to the first modal natural period of the soil profile in the assumed rigid half-space;
(2) for a two-layer soil profile in an elastic half-space, the fundamental period can be affected by radiation damping, and the effect is most significant at the turning point. When T_2/T_1 is smaller than the turning point, the fundamental period considering radiation damping is nearly equal to that of the first soil layer. In other cases, the fundamental period considering radiation damping is approximately equal to that ignoring radiation damping;
(3) for general multilayer soil profiles on bedrock, the fundamental period can be affected by radiation damping. When the impedance ratio of the lowest soil layer respect to the bedrock is large, and there are two adjacent soil layers with a large contrast, the fundamental period of the

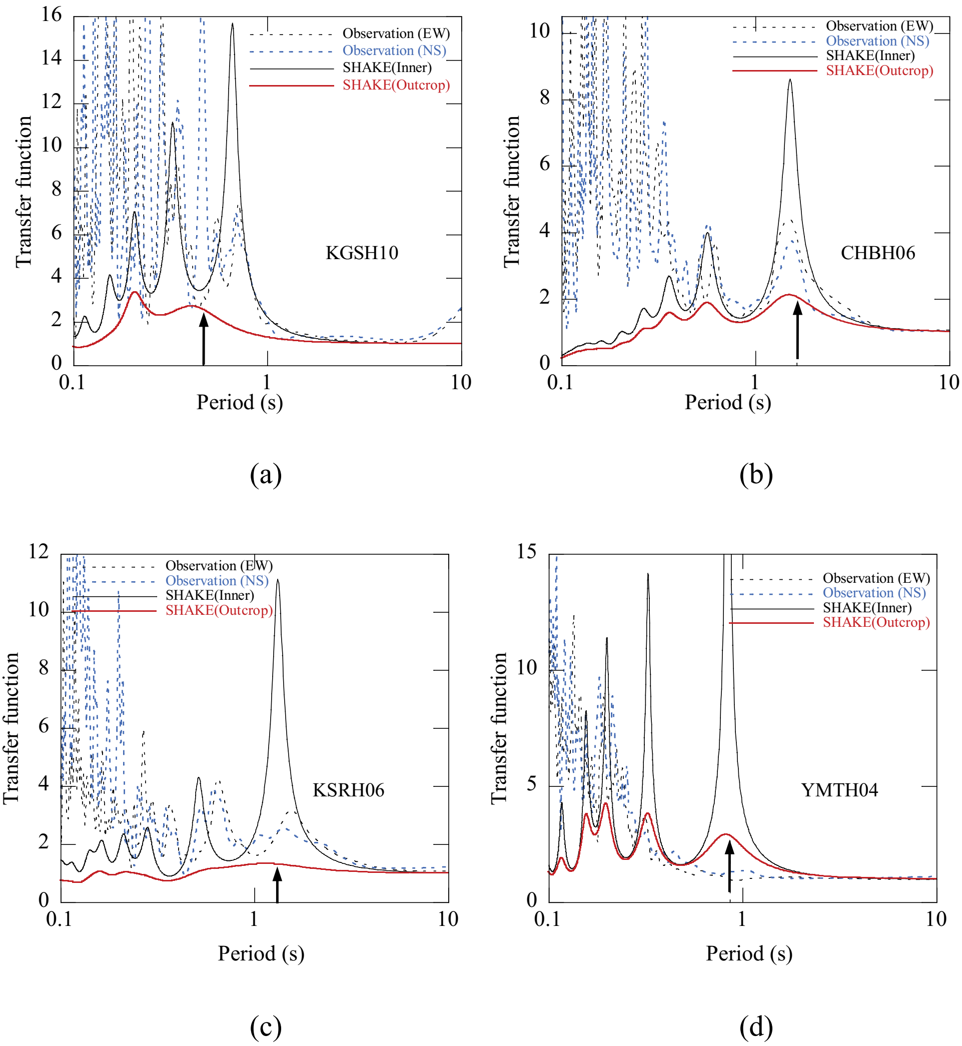

Figure 14. Comparison of transfer functions from SHAKE analyses and observations for the sites (a) KGSH10, (b) CHBH06, (c) KSRH06, and (d) YMTH04.

total soil profile may be shortened to that of the soil profile upon the interface of the two soil layers by radiation damping;

(4) the site fundamental periods by the proposed method agree very well with those from the SHAKE program.

Acknowledgments

The study is partially supported by the National Natural Science Foundation of China (Grant No. 51738001). The support is gratefully acknowledged.

Funding

This work was supported by the National Natural Science Foundation of China [51738001].

Declaration Of Interest Statement

There are no conflicts of interest to declare.

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