RELIABILITY EVALUATION OF PROJECT COMPLETION TIME USING FOURTH-MOMENT NORMAL TRANSFORMATION

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The reliability evaluation of project completion time is important in project management. Existing methods require finding the representative paths and complex calculations of mutual correlation coefficients between each pair of paths and the joint failure probability of each pair of representative paths. Herein, a simple and effective method for evaluating the reliability of the project completion time based on the fourth-moment normal transformation is proposed. The proposed method comprises three procedures: first, the overall performance function of the project completion time is established; second, the bivariate-dimension reduction integration is used to evaluate the first four moments of the performance function of the project completion time; third, the reliability of the project completion time is estimated by the fourth-moment normal transformation. The efficiency and accuracy of the proposed method are illustrated through an actual industrial example compared with the results from Monte Carlo simulations.

Keywords*: *Project completing time, Reliability evaluation, Fourth-moment normal transformation, Bivariate-dimension reduction

1. INTRODUCTION

The reliability evaluation of project completion time is important in project management. Most previous studies have only considered one critical path when evaluating the reliability of the project completion time[1),2)]. However, the evaluation results yielded by these methods are often inaccurate[3)~6)]; mainly because they disregard the correlation between network paths, which has been proven to significantly affect the reliability of the project completion time[7),8)].

Therefore, to consider the influence of all network paths, many methods have been proposed to evaluate the reliability of project completion time, including Monte Carlo simulations (MCSs)[4)~6),9),10)] and system reliability methods. MCSs require complicated calculations and can only provide numerical solutions, by which parametric analysis cannot be conducted. System reliability methods, such as narrow reliability bounds (NRB)[11),12)], generally require reliability bounds to be calculated and require complex calculations of the correlation coefficient between each pair of paths and the joint failure probability of each pair of paths.

Recently, Li et al.[8)] proposed a reliability evaluation method known as fast and accurate reliability bounds (FARB), which simplifies the computational complexity of existing methods by removing insignificant paths[8)]. However, it requires to find the representative paths, and the correlation coefficients between each pair of paths and the joint failure probability of each pair of representative paths must be calculated. Here, activities represent the process from one event (the ending point of the activity) to another; paths are the sequence of events and activities in the network. In FARB method, at least five to ten activities are required in each path to make the central limit theorem applicable, such that the duration of each path can be approximately to a normal distribution[8)]. Therefore, a simple and effective structural reliability method is necessary to evaluate the reliability of the project completion time.

This study applies a structural reliability method based on the fourth-moment normal transformation to evaluate the reliability of project completion time. Previous studies are reviewed in Section 2. In Section 3, after establishing the overall performance function of the project completion time, bivariate-dimension reduction integration is used to evaluate the first four moments of the overall performance function. Subsequently, the reliability of the project completion time is estimated using the fourth-moment normal transformation. In Section 4, to illustrate the efficiency and accuracy of the proposed method for evaluating the reliability of the project completion time, we present an actual industrial project and conduct an MCS for comparison.

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2. PREVIOUS REVIEW

Generally, a project often includes many paths. To consider the impact of all network paths on the reliability evaluation of project completion time, the project network should be considered as a series system. For a single series system, the project completion time being greater than the target duration is the union of all possible failure paths. Therefore, the failure probability of the project can be expressed as

$$P_F = P(f_1 \cup f_2 \cup ... \cup f_L) \tag{1}$$

where P = Prob, f_i denotes the case in which the duration of path i is greater than the target duration, and L denotes the number of paths in the project network.

Rather than directly calculating Eq. (1), which is considered to be complicated, some studies have used system methods to express the upper and lower bounds of P_F.

According to the narrow reliability bounds (NRB) method[11], the upper and lower bounds of P_F of the project completion time can be obtained as follows

$$P_{F_{LB}} = \left\{ P(f_1) + \sum_{i=2}^{L} \text{Max}\left[0, P(f_i) - \sum_{j=1}^{i-1} P(f_i \cap f_j) \right] \right\} \tag{2a}$$

$$P_{F_{LB}} = \left\{ P(f_1) + \sum_{i=2}^{L} \left[P(f_i) - \underset{j<i}{\text{Max}}\, P(f_i \cap f_j) \right] \right\} \tag{2b}$$

where the subscripts UB and LB indicate the upper and lower bounds, respectively; and $P(f_i \cap f_j)$ denotes the probability that the durations of paths i and j are both greater than the target duration; f_i and f_j denote the possible failure paths. $P(f_1) > P(f_2) > \ldots > P(f_L)$, $i = 1, \ldots, L$ indicates the number of network paths.

However, large-scale projects with many paths require excessive computations because the reliability of the project completion time estimated using Eqs. (2a)–(2b) considers the upper and lower bounds of $P(f_i \cap f_j)$ for any pair of paths in the network[8].

To overcome this limitation, Li et al.[8] proposed the FARB method. In this method, the insignificant paths are truncated based on the principle proposed by Ang et al.[8][13]: the paths in the network with high mean durations and high variances have a greater impact on the reliability of project completion time; if the durations on multiple paths are highly correlated, these paths will be replaced by the single representative path with the highest variance in each group of correlated paths; paths with low correlation coefficients are considered independent and are grouped as other representative paths.

The correlation coefficients between two paths can be calculated using Eq. (3a). According to Ang et al.[13], if the activities of the project network are assumed to be statistically independent, the correlation coefficient $\rho_{z,z'}$ between activity (z) and (z') is 0, so that the second term in Eq. (3b) becomes zero, and the covariance between the two paths can be obtained by using only the variance (σ_z^2) of duration of common activities between the two paths, as shown in Eq. (3c):

$$\rho_{ij} = \frac{\text{Cov}_{i,j}}{\sigma_i \sigma_j} \tag{3a}$$

$$\text{Cov}_{i,j} = \sum_{i=1}^{Z} a_{i,z} a_{j,z} \sigma_z^2 + \sum_{i=1}^{Z} \sum_{\substack{z=1 \\ z=z'}}^{Z} a_{i,z} a_{j,z} \rho_{z,z'} \sigma_z \sigma_{z'} \tag{3b}$$

$$\text{Cov}_{i,j} = \sum_{i=1}^{Z} a_{i,z} a_{j,z} \sigma_z^2 \tag{3c}$$

where $\text{Cov}_{i,j}$ is the covariance between paths i and j; σ_i and σ_j are the standard deviations of the duration of paths i and j, respectively; Z is the total number of activities; and $a_{i,z} = 1$ when activity (z) is contained in path (i), otherwise $a_{i,z} = 0$.

The correlation coefficient ρ_{ij} for each pair of paths i and j is compared with $\rho_0 = 0.5$[13]. When $\rho_{ij} > \rho_0$, paths i and j are termed as dependent, and the most significant path is selected as a representative path to represent these pairs of highly correlated paths; when $\rho_{ij} < \rho_0$, paths i and j are termed as independent[8].

When paths i and j are two independent and representative paths, the probability of any pair of f_i and f_j both occurring can be described as

$$P(f_i \cap f_j) = P(f_i)P(f_j) \tag{4}$$

Based on Eqs. (2a)–(2b) and Eq. (4), the upper and lower bounds of the reliability of the project completion time can be obtained as follows[8]

$$P_{S_{UB}} = 1 - \left\{ P(f_1) + \sum_{i=2}^{L} \text{Max}\left[0, P(f_i) - \sum_{j=1}^{i-1} P(f_i)P(f_j) \right] \right\} \tag{5a}$$

$$P_{S_{LB}} = 1 - \left\{ P(f_1) + \sum_{i=2}^{L} \left[P(f_i) - \underset{j<i}{\text{Max}}\, P(f_i)P(f_j) \right] \right\} \tag{5b}$$

The duration of each path is approximately to a normal distribution according to central limit theory[8]. Subsequently, for independent and representative paths, the probability of the project completion time being within the target duration t can be expressed as

$$P_{S_{UB}} = 1 - \left\{ \Phi(\beta_1) + \sum_{i=2}^{L} \text{Max}\left[0, \Phi(\beta_i) - \sum_{j=1}^{i-1} \Phi(\beta_i)\Phi(\beta_j) \right] \right\} \tag{6a}$$

$$P_{S_{LB}} = 1 - \left\{ \Phi(\beta_1) + \sum_{i=2}^{L} \left[\Phi(\beta_i) - \underset{j<i}{\text{Max}} \left\{ \Phi(\beta_i)\Phi(\beta_j) \right\} \right] \right\} \tag{6b}$$

where $\Phi(\bullet)$ indicates the cumulative distribution function (CDF) of the standard normal variable N (0,1), and β_i denotes the reliability index of the project completion time for path i, which can be expressed as[8]

$$\beta_i = \frac{\mu_i - t}{\sigma_i} \tag{7}$$

where μ_i is the mean of the completion time of path i.

Although the FARB method simplifies the computation compared with the NRB method, it requires the representative paths and the calculations of the correlation coefficient between each pair of paths and the joint failure probability of each pair of representative paths. The calculation of these factors is cumbersome in current project completion time evaluation methods. Therefore, a simple and efficient method must be developed to evaluate the reliability of project completion time.

3. PROPOSED METHOD FOR RELIABILITY EVALUATION OF PROJECT COMPLETION TIME

To avoid computing the correlation coefficients between each pair of paths and the joint failure probability between each pair of representative paths, an overall performance function of the entire project network is first established from the definition of failure probability. Subsequently, with the obtained first four moments of the performance function, the reliability of the project completion time can be finally evaluated by the fourth-moment normal transformation.

3.1 Establishing overall performance function of project completion time

For the project network, the failure probability of the project completion time P_F, as shown in Eq. (1), can be regarded as the union of all the failure paths. Each failure path f_i can be determined by a performance function $g_i = g_i(\mathbf{X})$, such that $f_i = (g_i \le 0)$. Thus, the failure probability of project completion time P_F can be rewritten as follows[14)]

$$\begin{aligned} P_F &= P(f_1 \cup f_2 \cup ... \cup f_L) \\ &= P[(g_1(\mathbf{X}) \le 0) \cup (g_2(\mathbf{X}) \le 0),..., \cup (g_L(\mathbf{X}) \le 0)] \end{aligned} \tag{8}$$

where $\mathbf{X} = \{x_1, x_2, …, x_n\}$ is the vector of random variables, which is referred to the vector of the activity durations of the project network in this paper; $g_i(\mathbf{X}) = t - T_i(\mathbf{X})$, of which t indicates the target project duration, and $T_i(\mathbf{X})$ is the duration of path i.

Meanwhile, the reliability of project completion time is the probability that no possible failure paths will occur, which can be expressed as

$$\begin{aligned} P_S &= P[\overline{f_1 \cup f_2 \cup ... \cup f_L}] = P[\overline{f_1} \cap \overline{f_2} \cap ... \cap \overline{f_L}]] \\ &= P[(g_1(\mathbf{X}) > 0) \cap (g_2(\mathbf{X}) > 0) \cap ... \cap (g_L(\mathbf{X}) > 0)] \end{aligned} \tag{9}$$

Eq. (9) indicates that the reliability of the project completion time is the event that all L performance functions are larger than zero, which means that the target duration t is larger than the maximum of $T_i(\mathbf{X})$, which can be rewritten as

$$P_S = P\{t - \max[T_1(\mathbf{X}), T_2(\mathbf{X}), ..., T_L(\mathbf{X})] > 0\} \tag{10}$$

The corresponding failure probability of the project completion time can be obtained as

$$P_F = P\{t - \max[T_1(\mathbf{X}), T_2(\mathbf{X}), ..., T_L(\mathbf{X})] \le 0\} \tag{11}$$

Therefore, the overall state performance function of the project completion time, $G(\mathbf{X})$, can be expressed as

$$G(\mathbf{X}) = t - \max[T_1(\mathbf{X}), T_2(\mathbf{X}), ..., T_L(\mathbf{X})] \tag{12}$$

3.2 Evaluating first four moments of overall performance function

The computation of the reliability of project completion time implicates the evaluation of the first four moments of the project completion time performance function. Using the point-estimate method in independent standard normal space, the first four moments of $G(\mathbf{X})$ can be estimated as follows[14),15)]

$$\mu_G = \sum \prod_{i=1}^{n} P_{ci} \left\{ G\left[T^{-1}(u_{c1}, ..., u_{ci}, ..., u_{cn}) \right] \right\} \tag{13}$$

$$\sigma_G^2 = \sum \prod_{i=1}^{n} P_{ci} \left\{ G\left[T^{-1}(u_{c1}, ..., u_{ci}, ..., u_{cn}) \right] - \mu_G \right\}^2 \tag{14}$$

$$\alpha_{3G}\sigma_G^3 = \sum \prod_{i=1}^{n} P_{ci} \left\{ G\left[T^{-1}(u_{c1}, ..., u_{ci}, ..., u_{cn}) \right] - \mu_G \right\}^3 \tag{15}$$

$$\alpha_{4G}\sigma_G^4 = \sum \prod_{i=1}^{n} P_{ci} \left\{ G\left[T^{-1}(u_{c1}, ..., u_{ci}, ..., u_{cn}) \right] - \mu_G \right\}^4 \tag{16}$$

where n denotes the dimension of random vector $\mathbf{X}$; c indicates the distinct combination of n items from the group [1, 2, …, m]; m denotes the number of estimating points; ci denotes the ith term of c; u_{ci} and P_{ci} indicate the cith estimating points and the corresponding weights, respectively; μ_G, σ_G, α_{3G}, and α_{4G} denote the first four moments, i.e., mean, standard deviation, skewness, and kurtosis of $G(\mathbf{X})$, respectively, and T^{-1} denotes the Rosenblatt transformation.

Utilizing Eqs. (13)–(16), m^n function evaluations are necessary to determine $G(\mathbf{X})$; at large values of n, the calculation becomes heavy. Thus, the dimension reduction integration[16)] is adopted to simplify the calculation. Considering the first four moments of $G(\mathbf{X})$, the bivariate-dimension reduction method[16)] is introduced. The performance function $G(\mathbf{X})$ can then be approximated by $G^*(\mathbf{X})$ as

$$\begin{aligned} G(\mathbf{X}) &\cong G^*(\mathbf{X}) = G^*[T^{-1}(\mathbf{U})] \\ &= \sum_{i<j} G_{i,j} - (n-2)\sum_{i=1}^{n} G_i + \frac{(n-1)(n-2)}{2} G_0 \end{aligned} \tag{17}$$

where

$$G_{i,j} = G\left[\mu_1, ..., T^{-1}(u_i), ..., T^{-1}(u_j), ..., \mu_n \right] \tag{18}$$

$$G_i = G\left[\mu_1, ..., T^{-1}(u_i), ..., \mu_n \right] \tag{19}$$

$$G_0 = G(\mu_1, ..., \mu_i, ..., \mu_n) \tag{20}$$

where $G_{i,j}$ indicates a two-dimensional function in terms of $T^{-1}(u_i)$ and $T^{-1}(u_j)$, i, j = 1, …, n, and $i < j$; G_i is a one-dimensional function of $T^{-1}(u_i)$, G_0 denotes a constant; μ_i (i = 1, …, n) denotes the mean value of random variables. The deviation can be found in reference 16.

Therefore, the kth raw moments of $G(\mathbf{X})$ can be approximately formulated as

$$\begin{aligned} \mu_{kG} &= E\left\{ \left[G(\mathbf{X}) \right]^k \right\} \cong E\left\{ \left[G^*(\mathbf{X}) \right]^k \right\} = E\left\{ \left\{ G^*[T^{-1}(\mathbf{U})] \right\}^k \right\} \\ &\cong \sum_{i<j} \mu_{G_{i,j}}^k - (n-2)\sum_{i=1}^{n} \mu_{G_i}^k + \frac{(n-1)(n-2)}{2} G_0^k \end{aligned} \tag{21}$$

where

$$G_0^k = \left[G(\mu_1, ..., \mu_i, ..., \mu_n) \right]^k \tag{22}$$

$$\mu_{G_i}^k = \int_{-\infty}^{\infty} \left\{ G_i\left[\mu_1, ..., T^{-1}(u_i), ..., \mu_n \right] \right\}^k \phi(u_i) du_i \tag{23}$$

$$\mu_{G_{i,j}}^k = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ G_{i,j}\left[\mu_1, ..., T^{-1}(u_i), ..., T^{-1}(u_j), ..., \mu_n \right] \right\}^k \phi(u_i)\phi(u_j) du_i du_j \tag{24}$$

and μ_{kG} (k = 1,…,4) are the first four raw moments of $G(\mathbf{X})$.

Utilizing the point-estimate method[15)], the one-dimensional integral in Eq. (23) can be approximated as follows

$$\mu_{G_i}^k = \sum_{r=1}^{m} P_r \left\{ G_i\left[\mu_1, ..., T^{-1}(u_{ir}), ..., \mu_n \right] \right\}^k \tag{25}$$

where m denotes the number of estimated points; $u_{ir} = \sqrt{2}\, x_r$ and $P_r = w_r / \sqrt{\pi}$ denote the estimated points and corresponding weights, respectively, in which x_r and w_r are the abscissas and weights of the

Gauss–Hermite quadrature with weight function $\exp(-x^2)$[17].

Similarly, the two-dimensional integral in Eq. (24) can be estimated as

$$\mu_{G_{i,j}}^{k}=\sum_{r_1=1}^{m}\sum_{r_2=1}^{m}P_{r_1}P_{r_2}\left\{G_{i,j}\left[(\mu_1,...,T^{-1}(u_{ir_1}),...,T^{-1}(u_{jr_2}),...,\mu_n)\right]\right\}^{k} \tag{26}$$

Based on the five-point estimate method (m = 5) proposed by Zhao and Ono[15], the value of u_{iq} ($q = r$, r_1, r_2) and weight P_r are obtained as

$$u_{i1}=-2.8569700,\ P_1=1.12574\times10^{-2} \tag{27}$$

$$u_{i2}=-1.3556262,\ P_2=0.2220759 \tag{28}$$

$$u_{i3}=0,\ P_3=0.5333 \tag{29}$$

$$u_{i4}=1.3556262,\ P_4=0.2220759 \tag{30}$$

$$u_{i5}=2.8569700,\ \ P_5=1.12574\times10^{-2} \tag{31}$$

Finally, the first four moments of the performance function (μ_G, σ_G, α_{3G}, and α_{4G}), described in Eq. (12), can be estimated as follows

$$\mu_G=\mu_{1G} \tag{32}$$

$$\sigma_G=\sqrt{\mu_{2G}-\mu_{1G}^2} \tag{33}$$

$$\alpha_{3G}=\left(\mu_{3G}-3\mu_{2G}\mu_{1G}+2\mu_{1G}^3\right)/\sigma_G^3 \tag{34}$$

$$\alpha_{4G}=\left(\mu_{4G}-4\mu_{3G}\mu_{1G}+6\mu_{2G}\mu_{1G}^2-3\mu_{1G}^4\right)/\sigma_G^4 \tag{35}$$

Utilizing the five-point estimation method based on bivariate-dimension reduction integral to calculate the first four moments of the performance function, only ($C_n^2\times5^2+C_n^1\times5+1$) times are required.

3.3 Estimating the reliability of the project completion time using fourth-moment normal transformation

After the first four central moments are obtained, the reliability of the project completion time can be estimated using the fourth-moment normal transformation.

For a random variable G, i.e., if its first four moments μ_G, σ_G, α_{3G}, and α_{4G} are known, the standardized random variable G_s can be expressed by the fourth-moment normal transformation as follows [18]:

$$G_s=(G-\mu_G)\ /\ \sigma_G=S_u(u)=a_1+a_2u+a_3u^2+a_4u^3 \tag{36}$$

where G_s is a standardized random variable, with the skewness and kurtosis being the same as those of G, $S_u(u)$ is a third-order polynomial of u, a_1, a_2, a_3, and a_4 are coefficients calculated by setting the first four moments of the left side of Eq. (36) equal to those of the right side. A detailed solution process is available from the study of Zhao et. al.[19], and it is listed in the Appendix. Therefore, the inverse function of the relationship between the standard normal variable u and the standardized variable G_s can be expressed as

$$u=S^{-1}(G_s) \tag{37}$$

where S^{-1} denotes the inverse function of S.

The third-order polynomial in Eq. (36) has many solutions. In order to obtain the unique solution, the explicit expressions of u are divided into six types according to the applicable range of G, and the explicit expressions of u are summarized in Table 1 according to Zhao et al.[19]. For most combinations of α_{3G}, and α_{4G}, the following expression is generally applicable.

$$u=\sqrt[3]{A}+\sqrt[3]{B}-a/3 \tag{38}$$

Table 1 Complete monotonic expressions of u related to G_s

Parameters	Range of G	Expression of u	Type
$a_4<0$	$J_2^*<G<J_1^*$	$-2r\cos[(\theta+\pi)/3]-a/3$	I
$a_4>0$ $P<0$ $\alpha_{3G}\geq0$	$J_1^*<G<J_2^*$	$2r\cos(\theta/3)-a/3$	II
	$G\geq J_2^*$	$\sqrt[3]{A}+\sqrt[3]{B}-a/3$	
$\alpha_{3G}<0$	$J_1^*<G<J_2^*$	$-2r\cos[(\theta-\pi)/3]-a/3$	III
	$G\leq J_1^*$	$\sqrt[3]{A}+\sqrt[3]{B}-a/3$	
$P\geq0$	$(-\infty,\infty)$	$\sqrt[3]{A}+\sqrt[3]{B}-a/3$	VI
a_4=0 $\alpha_{3G}\neq0$	$a_2^2+4a_3(a_3+G_s)\geq0$	$[-a_2+\sqrt{a_2^2+4a_3(a_3+G_s)}]/2a_3$	V
$\alpha_{3G}=0$	$(-\infty,\infty)$	G_s	IV

Then, based on Eqs. (39)–(44), the parameters p, r, θ, a, A, B, J_1^* and J_2^* of Table 1 can be obtained.

$$p=\frac{3a_2a_4-a_3^2}{3a_4^2} \tag{39}$$

$$\Delta=(\frac{p}{3})^3+(\frac{q}{2})^2,\ q=\frac{2a^3}{27}-\frac{ac}{3}-a-\frac{G_s}{a_4},\ a=\frac{a_3}{a_4},\ c=\frac{a_2}{a_4} \tag{40}$$

$$\theta=\arccos(\frac{-q}{2r^3}),\ r=\sqrt{-\frac{p}{3}} \tag{41}$$

$$A=-\frac{q}{2}+\sqrt{\Delta},\ B=-\frac{q}{2}-\sqrt{\Delta} \tag{42}$$

$$J_1^*=\sigma_Ga_4(-2r^3+\frac{2a^3}{27}-\frac{ac}{3}-a)+\mu_G \tag{43}$$

$$J_2^*=\sigma_Ga_4(2r^3+\frac{2a^3}{27}-\frac{ac}{3}-a)+\mu_G \tag{44}$$

According to Eq. (36) the CDF and PDF are expressed as[20]

$$F_{G_s}(G_S)=\Phi(u) \tag{45}$$

$$f(G_S)=\frac{\phi(u)}{\sigma_X(3a_4u^2+2a_3u+a_2)} \tag{46}$$

Substituting Eq. (36) with an explicit expression of u listed in Table 1 into Eq. (45) yields

$$F_{G_s}(G_S)=\Phi(u)=\Phi[S^{-1}(G_S)] \tag{47}$$

Therefore, the reliability of G can be expressed as

$$P_S=1-P_F=1-P[G\leq0]=1-F_{G_s}(\frac{-\mu_G}{\sigma_G})=1-\Phi[S^{-1}(\frac{-\mu_G}{\sigma_G})] \tag{48}$$

4. EXAMPLE

In this section, an industrial building project involving the construction of a single storey industrial building with an adjoining parking lot is analyzed. This project, which was first investigated by

Brend et al.[21], comprises four parts: reinforced concrete piers, frost walls, structural steel columns, and a precast roof deck. There are 33 paths and 38 activities related to these four parts. The descriptions and statistical information of the activities are listed in Table 2. There are 8 activities with zero mean and zero variance, which are the dummy activities. Due to that the project arrow diagram must connect all nodes except the beginning and the end, the dummy activities are connected to the nodes of the activities, and the next activity can only move forward after the previous activity completed. The corresponding project network is shown in Fig.1, where the number of ○ is the node number and an activity occurs between two nodes. And all the paths of this project network with the corresponding activities are listed in Table 3. More details regarding the various activities are available in the study of Guo et.al.[22] due to space limitation of the paper.

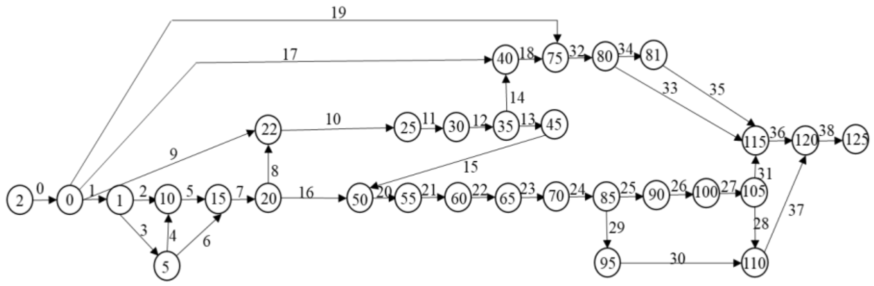

Fig.1 An industrial building project network

Table 2 Statistical data of activity durations (days)

Node	Activity No.	Descriptions of activities	μ	σ	Distribution
2-0	0	Dummy	0	0	Lognormal
0-1	1	Mobilisation	32	3.2	Lognormal
1-10	2	Move in	2	0.5	Lognormal
1-5	3	Initial layout	2	0.5	Lognormal
5-10	4	Dummy	0	0	Lognormal
10-15	5	Site rough grading	2	0.5	Lognormal
5-15	6	Layout of piers	1	0.5	Lognormal
15-20	7	Excavate piers	2	1	Lognormal
20-22	8	Dummy	0	0	Lognormal
0-22	9	Order and deliver rebars	40	12	Lognormal
22-25	10	Form and rebars piers	2	0.5	Lognormal
25-30	11	Pour piers	2	0.5	Lognormal
30-35	12	Cure piers	4	0.8	Lognormal
35-45	13	Strip piers	1	0.1	Lognormal
35-40	14	Dummy	0	0	Lognormal
45-50	15	Dummy	0	0	Lognormal
20-50	16	Excavate frost walls	1	0.5	Lognormal
0-40	17	Order and deliver structural steel columns	60	12	Lognormal
40-75	18	Erect structural steel columns	5	1	Lognormal
0-75	19	Order and deliver precast roof deck	30	6	Lognormal
50-55	20	Form and mesh frost walls	3	0.9	Lognormal
55-60	21	Pour frost walls	1	0.3	Lognormal
60-65	22	Cure frost walls	4	0.4	Lognormal
65-70	23	Strip frost walls	1	0.1	Lognormal
70-85	24	Backfill	2	0.5	Lognormal
85-90	25	Grade and compact gravel for floor	2	0.2	Lognormal
90-100	26	Rebar floor and set screeds	2	0.5	Lognormal
100-105	27	Pour and finish floor	2	0.5	Lognormal
105-110	28	Dummy	0	0	Lognormal
85-95	29	Excavate, grade parking	2	0.2	Lognormal
95-110	30	Stone base for parking	1	0.2	Lognormal
105-115	31	Dummy	0	0	Lognormal
75-80	32	Set roof deck	5	1.5	Lognormal
80-115	33	Hang siding and waterproof roof	6	1.2	Lognormal
80-81	34	Dummy	0	0	Lognormal

Table 2 Continued

Node	Activity No.	Descriptions of activities	μ	σ	Distribution
81-115	35	Hang doors	4	1.2	Lognormal
115-120	36	Clean up	2	0.5	Lognormal
110-120	37	Bituminous surface in parking	3	0.3	Lognormal
120-125	38	Dummy	0	0	Lognormal

Table 3 Activity network paths of the industrial project

Path	Activities of each path
1	19,32,34,35,36
2	19,32,33,36
3	17,18,32,33,36
4	17,18,32,34,35,36
5	9,10,11,12,14,18,32,34,35,36
6	9,10,11,12,14,18,32,33,36
7	9,10,11,12,13,15,20,21,22,23,24,25,26,27,31,36
8	9,10,11,12,13,15,20,21,22,23,24,25,26,27,28,37
9	9,10,11,12,13,15,20,21,22,23,24,29,30,37
10	1,2,5,7,8,10,11,12,14,18,32,34,35,36
11	1,2,5,7,8,10,11,12,14,18,32,33,36
12	1,2,5,7,8,10,11,12,13,15,20,21,22,23,24,25,26,27,31,36
13	1,2,5,7,8,10,11,12,13,15,20,21,22,23,24,25,26,27,28,37
14	1,2,5,7,8,10,11,12,13,15,20,21,22,23,24,29,30,37
15	1,2,5,7,16,20,21,22,23,24,25,26,27,31,36
16	1,2,5,7,16,20,21,22,23,24,25,26,27,28,37
17	1,2,5,7,16,20,21,22,23,24,29,30,37
18	1,3,4,5,7,8,10,11,12,14,18,32,34,35,36
19	1,3,4,5,7,8,10,11,12,14,18,32,33,36
20	1,3,4,5,7,8,10,11,12,13,15,20,21,22,23,24,25,26,27,31,36
21	1,3,4,5,7,8,10,11,12,13,15,20,21,22,23,24,25,26,27,28,37
22	1,3,4,5,7,8,10,11,12,13,15,20,21,22,23,24,29,30,37
23	1,3,4,5,7,16,20,21,21,22,23,24,25,26,27,31,36
24	1,3,4,5,7,16,20,21,22,23,24,25,26,27,28,37
25	1,3,4,5,7,16,20,21,22,23,24,29,30,37
26	1,3,6,7,8,10,11,12,14,18,32,34,35,36
27	1,3,6,7,8,10,11,12,14,18,32,33,36
28	1,3,6,7,8,10,11,12,13,15,20,21,22,23,24,25,26,27,31,36
29	1,3,6,7,8,10,11,12,13,15,20,21,22,23,24,25,26,27,28,37
30	1,3,6,7,8,10,11,12,13,15,20,21,22,23,24,29,30,37
31	1,3,6,7,16,20,21,21,22,23,24,25,26,27,31,36
32	1,3,6,7,16,20,21,22,23,24,25,26,27,28,37
33	1,3,6,7,16,20,21,22,23,24,29,30,37

The overall performance function $G(\mathbf{X})$ of the project completion time can be defined according to Eq. (12) as

$$G(\mathbf{X}) = t - \max[T_1(\mathbf{X}),...,T_{33}(\mathbf{X})] \tag{49}$$

Based on Eq. (17), $G(\mathbf{X})$ in Eq. (49) can be approximated as

$$G(\mathbf{X}) \cong G^*(\mathbf{X}) = G^*[T^{-1}(\mathbf{U})] = \sum_{i<j} G_{i,j} - 28\sum_{i=1}^{30} G_i + 420G_0 \tag{50}$$

For different target durations t, by substituting all the means of the random variables in Table 2 into Eq. (22), G_0^k (k = 1, 2, 3, 4) can be obtained.

$$\begin{aligned} G_0^k &= \left[G_\mu(\mu_1,...,\mu_i,...,\mu_{30})\right]^k \\ &= \left\{t - \max[T_1(\boldsymbol{\mu}),...,T_{33}(\boldsymbol{\mu})]\right\}^k = (t-82.041)^k \end{aligned} \tag{51}$$

Using the five estimate points in standard normal space as expressed by Eqs. (27)–(31) and the statistical information of each random variable in Table 2, the corresponding original space estimate points of random variables can be obtained via the inverse Rosenblatt transformation.

Substituting the original space five estimate points of G_i (i = 1, …, 30) into Eq. (25) and Eq. (26), the value of $\mu_{G_i}^k$ and μ_{Gij}^k ($i < j$) can be steadily obtained. Then, with the aid of Eq. (21), the value of u_{kG} can be obtained. Ultimately, utilizing the obtained first four raw moments, the first four moments of $G(\mathbf{X})$ can be obtained as

$$\mu_G = \mu_{1G} = t - 82.041 \tag{52}$$

$$\sigma_G = \sqrt{\mu_{2G} - \mu_{1G}^2} = 10.826 \tag{53}$$

$$\alpha_{3G} = \left(\mu_{3G} - 3\mu_{2G}\mu_{1G} + 2\mu_{1G}^3\right)/\sigma_G^3 = -0.974 \tag{54}$$

$$\alpha_{4G} = \left(\mu_{4G} - 4\mu_{3G}\mu_{1G} + 6\mu_{2G}\mu_{1G}^2 - 3\mu_{1G}^4\right)/\sigma_G^4 = 4.423 \tag{55}$$

Using the first four moments of $G(\mathbf{X})$, the coefficients defined in Eq. (36) can be obtained as a_1 = 0.159, a_2 = 0.956, a_3 = –0.159, and a_4 = 0.006. Then based on Eq. (39), the value of P can be obtained

$$p = \frac{3a_2a_4 - a_3^2}{3a_4^2} = -64.753 \tag{56}$$

Because $a_4 > 0$, $P < 0$ and $\alpha_{3G} < 0$, the expression of u belongs to Type III of Table 1. With Eqs. (40)–(44), the parameters r, θ, a, A, B, J_1^* and J_2^* can be calculated

$$\Delta = 3.05784\times10^5 - 8343.77t + 55.106t^2,\ q = -1123.961 + 14.847t \tag{57a}$$

$$a = -25.593,\ c = -1192.448 + 14.847t \tag{57b}$$

$$\theta = \arccos(5.604 - 0.074t),\ r = 4.646 \tag{58}$$

$$A = 561.996 - 7.423t + \sqrt{3.05784\times10^5 - 8343.77t + 55.106t^2} \tag{59a}$$

$$B = 561.996 - 7.423t - \sqrt{3.05784\times10^5 - 8343.77t + 55.106t^2} \tag{59b}$$

$$J_1^* = 294.607,\quad J_2^* = -106.508 \tag{60}$$

According to Eq. (48), the reliability of the project completion time under different target durations t can be obtained, as presented in Fig.2.

$$P_S = 1 - P_F = 1 - F_{G_s}\left(\frac{82.041-t}{10.826}\right) = 1 - \Phi\left[S^{-1}\left(\frac{82.041-t}{10.826}\right)\right] \tag{61}$$

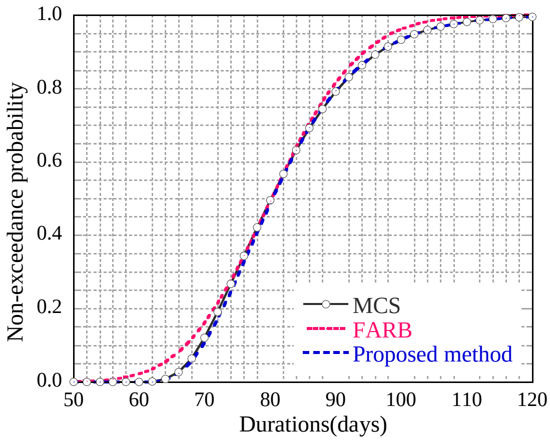

Fig.2 Results comparison between different methods

Fig.2 presents the results of the reliability of the project completion time obtained using the proposed method, FARB method and the MCS with 1,000,000 samples, respectively. While, the proposed method can generate results with only $C_{30}^2\times5^2+C_{30}^1\times5+1=$ 11026 times, resulting in significantly less computation than that associated with MCS. If the number of activities is fewer, the efficiency will be more significantly.

As shown, the results obtained using the proposed method agree well with those obtained using MCS. Furthermore, unlike the FARB method, the proposed method does not require the calculations of the correlation coefficients between any pair of paths and the joint failure probability of any pair of representative paths.

5. CONCLUSIONS

We presented a simple and effective method using fourth-moment normal transformation to evaluate the reliability of the project completion time. The proposed method does not require calculations of the correlation coefficient between any pair of paths and the joint failure probability of any pair of paths. A numerical example demonstrated the accuracy and efficiency of the proposed method. It is shown that the structural reliability method can be effectively applied to evaluate the reliability of the project completion time. It should be noted that some activities have correlations; this will be explored in future study.

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APPENDIX. COMPUTATION FOR THE FOUR COEFFICIENTS

Following Fleishman[19], the four coefficients of a_1, a_2, a_3 and a_4 of Eq. (36) can be calculated by letting the first four moments (mean μ_{Gs}, standard deviation σ_{Gs}, skewness α_{3Gs}, and kurtosis α_{4Gs}) of the left side of Eq. (36) equal to the first four moments of $S_u(u)$ of the right side

$$\mu_{G_s} = \mu_{S_u(u)} = a_1 + a_3 = 0 \tag{62a}$$

$$\sigma_{G_s} = \sigma_{S_u(u)} = a_2^2 + 2a_3^2 + 6a_2a_4 + 15a_4^2 = 1 \tag{62b}$$

$$\alpha_{3G_s} = \alpha_{3S_u(u)} = 6a_2a_3 + 8a_3^3 + 72a_2a_3a_4 + 270a_3a_4^2 \tag{62c}$$

$$\alpha_{4G_s} = \alpha_{4S_u(u)} = 3(a_2^4 + 20a_2^3a_4 + 210a_2^2a_4^2 + 3465a_4^4) + 12a_3^2(5a_2^2 + 5a_3^2 + 78a_2a_4 + 375a_4^2) \tag{62d}$$

By simplifying Eqs. (62a)–(62d), parameters a_2 and a_4 can be gotten as follows

$$\alpha_{3G_s}^2 = 2A_1A_2^2 \text{ , } \alpha_{4G_s} = 3A_1A_3 + 3A_4 \tag{62a}$$

where

$$A_1 = 1 - a_2^2 - 6a_2a_4 - 15a_4^2 \tag{62c}$$

$$A_2 = 2 + a_2^2 + 24a_2a_4 + 105a_4^2 \tag{62d}$$

$$A_3 = 5 + 5a_2^2 + 126a_2a_4 + 675a_4^2 \tag{62e}$$

$$A_4 = a_2^4 + 20a_2^3a_4 + 210a_2^2a_4^2 + 1260a_2a_4^3 \tag{62f}$$

Due to the values of α_{3G}, and α_{4G} are known, the parameters a_2 and a_4 can be obtained from Eqs. (62a)–(62f), which can be solved by a suitable nonlinear equation solver, with preconditions:

(1) The parameters are all real numbers;

(2) A_1 ($A_1 = 2\alpha_{3GS}^2$) is not smaller than zero;

(3) When $\alpha_{3Gs} = 0$, $\alpha_{4Gs} = 3$, $a_3 = a_4 = 0$ and $a_2 = 1$. (To make sure that the fourth-moment transformation consists of normal distribution.)

After the parameters a_2 and a_4 have been determined, the parameters a_1 and a_3 can be readily obtained as follows

$$a_3 = -a_1 = \frac{\alpha_{3G_s}}{2A_2} \tag{62g}$$