Journal of Earthquake and Tsunami
Vol. 13, No. 1 (2019) 1950003 (17 pages)

DOI: 10.1142/S1793431119500039

A Simple Approach for Estimating the Fundamental Mode Shape of Layered Soil Profiles

Haizhong Zhang* and Yan-Gang Zhao†

Department of Architecture
Kanagawa University 3-27-1 Rokkakubashi
Kanagawa-ku Yokohama 221-8686, Japan
**r201570165oj@jindai.jp*
†zhao@kanagawa-u.ac.jp

Received 15 January 2018
Accepted 24 October 2018
Published 10 January 2019

The fundamental mode shape of layered soil profiles is a key site response parameter, it has been adopted into the Japanese seismic code to represent the shape of the soil displacement response along the vertical direction. In this study, a simple approach for estimating the fundamental mode shape of layered soil profiles is developed. The proposed approach can directly model the fundamental mode shape and can be conveniently implemented using arithmetic operations, thus making it suitable to be used by the engineers. The assessments of the proposed approach using a series of layered soil profiles demonstrate that it can produce results in close agreement with the actual results.

Keywords: Layered soil profiles; the fundamental mode shape; site response.

1. Introduction

The natural frequencies, mode shapes, and participation factors are the basic information that are important to perform the site response analysis of layered soil profiles [Zhao, 1997; Sarma, 1994]. Given that the site response is generally dominated by the first mode, it is often approximated by the response of the first mode in terms of the fundamental period and mode shape [Francesca *et al.*, 2010; Tsang *et al.*, 2006; Lam *et al.*, 2001; Kenji *et al.*, 2001]. Under the Japanese seismic code, the non-linear site effects are estimated using the fundamental mode shape to represent the shape of the soil displacement response along the vertical (height) direction [Mistumasa *et al.*, 2003; MLIT, 2000]. In principle, the fundamental period and mode shape can be exactly obtained by solving the equilibrium equations of free vibration, or by performing an accurate estimation via eigenvalue analysis by discretizing the continuous soil profile into a lumped-parameter multi-degree-of-freedom (MDOF)

†Corresponding author.

model. However, the implementation of such methods is generally quite cumbersome in practice. A primary example of a situation in which a simple approach for determining the fundamental mode shape of layered soil profiles is desirable, involves the determination of the "base", e.g. the extent of the crust to be included in the model of a bottomless soil profile, before calculating the response. In such situations, understanding the mode shape of the soil model can be useful [Hadjian, 2002]; however, this modeling decision is usually performed in an ad-hoc fashion, and accurate computation is generally not desirable at this stage.

Although numerous studies have focused on simple approaches for obtaining the fundamental periods of layered soil profiles [Hadjian, 2002; Dobry *et al.*, 1976; Vijayendra *et al.*, 2015], there have been few studies on developing approaches for obtaining the fundamental mode shape. Dobry *et al.* [1976] concluded that the Simplified Version of the Rayleigh procedure can provide accurate solutions for both the fundamental period and mode shape. But, the simplified Rayleigh procedure is iterative [Hadjian, 2002]. Subsequently, Hadjian [2002] developed a direct approach to calculate the fundamental mode shape. However, this method is not readily applicable in practicing engineering because the implementation of the Hadjian method requires repeated exponential calculation.

In this paper, a simpler and accurate method for estimating the fundamental mode shape of layered soil profiles is proposed. The proposed approach can directly model the fundamental mode shape and can be conveniently implemented using arithmetic operations. The remainder of the paper is organized as follows. In Sec. 2, the simplest current method for estimating the fundamental mode shape, i.e. the Hadjian method [Hadjian, 2002], is briefly reviewed. In Sec. 3, a new equation for the analysis of the natural frequencies and mode shapes of layered soil profiles is derived. In Sec. 4, a new approach based on the use of this equation in conjunction with a simple equation for the fundamental period is developed to estimate the fundamental mode shape. In Sec. 5, the accuracy of the proposed procedure is assessed and compared with that of the Hadjian method using a series of layered soil profiles. It is observed that both methods provide considerably good accuracy while estimating the fundamental mode shape; however, the implementation of the proposed method is much more convenient than that of the Hadjian method. In Sec. 6, the conclusions are presented.

2. The Hadjian Method

Several simple methods for the estimation of the fundamental mode shape of layered soil profiles have been developed [Hadjian, 2002; Dobry *et al.*, 1976]. Among these methods, the method proposed by Hadjian [2002] is currently considered to be the simplest and can be implemented using spreadsheets. This section briefly reviews Hadjian's method.

The Hadjian method estimates the fundamental mode shape using the results of the fundamental period. Thus, before calculating the fundamental mode shape, one should estimate the fundamental period. The Hadjian method as an enhancement to

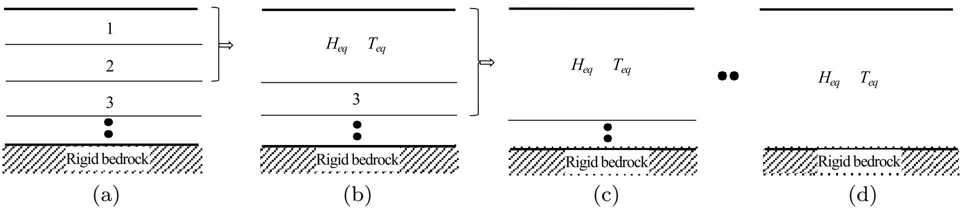

Fig. 1. Estimation of the fundamental period of a multilayer soil profile on bedrock using the Madera procedure.

the Madera procedure [Madera, 1970] calculates the fundamental period based on the solution of the fundamental period of a two-layer soil profile on bedrock. Figure 1 illustrates the calculation process that is followed in the Hadjian method. The solution starts by replacing the top two layers of an n-layer soil profile by an equivalent single layer using the two-layer system solution. This first equivalent "single" layer and the third layer of the n-layer profile are then treated as a second two-layer system, and, in turn, replaced by an equivalent single layer. By application of this procedure successively to the remaining lower layers of the soil profile, the solution of the fundamental period of the total soil profile is obtained. The Hadjian method uses the two-layer system solution successively a total of $n-1$ times. During the application of this procedure, the fundamental period of the soil layers that are decoupled from the ground surface to each soil interface, T_{1-i}, can also be obtained. Here, the subscript $1-i$ refers to the layers 1 through i.

The two-layer system solution was originally presented by Madera [1970] in the form of charts; subsequently, this solution was replaced by more convenient approximate equations by Hadjian [2002], which are as follows:

$$\frac{T}{T_1} = 1 + \frac{H_1}{H_2}\left(\frac{T_2}{T_1}\right)^2, \quad T_2/T_1 \leq 1, \tag{1}$$

$$\frac{T}{T_1} = \sqrt{\frac{\pi^2}{8}\left[0.75 + \left(\frac{T_2}{T_1}\right)^2\left(1 + 2\frac{H_1\rho_1}{H_2\rho_2}\right)\right]}, \quad H_1/H_2 > 1, \tag{2}$$

$$\frac{T}{T_1} = \left[1 + \beta\left(\frac{T_2}{T_1}\right)^\alpha\left(1 + \frac{H_1\rho_1}{H_2\rho_2}\right)^\alpha\right]^{\frac{1}{\alpha}}, \quad H_1/H_2 \leq 1, \tag{3}$$

where

$$\alpha = 4 - 1.8\frac{H_1\rho_1}{H_2\rho_2}, \quad \beta = 1 - 0.2\left(\frac{H_1\rho_1}{H_2\rho_2}\right)^2.$$

Here, H_1, ρ_1, and T_1 $(= 4H_1/V_1)$ are the height, density, and decoupled fundamental period of the top layer, respectively; H_2, ρ_2, and T_2 $(= 4H_2/V_2)$ are the height, density, and decoupled fundamental period of the bottom layer, respectively. Furthermore, V_1 and V_2 are the shear wave velocities of the layers, and T is the fundamental period of the two-layer soil profile on bedrock.

Further, using the results of the fundamental period, the fundamental mode shape, $X_1(z)$, can be estimated as follows:

$$X_1(z_i) = \cos\left(\frac{\pi}{2}\frac{T_{1-i}}{T_{1-n}}\right), \tag{4}$$

where $X_1(z_i)$ represents the value of the fundamental mode shape at the ith soil interface, T_{1-i} represents the decoupled fundamental period of the soil layers from the first interface (the ground surface) to the ith interface, and T_{1-n} represents the fundamental period of the overall soil profile. As mentioned above, all values of T_{1-i} $(i = 1 - n)$ can be obtained, while estimating the fundamental period using the Hadjian method.

After estimating the fundamental mode shape, the corresponding participation factor, p, can be obtained by

$$p = \frac{X_1^T\{m_i\}}{X_1^T[M]X_1}, \tag{5}$$

where $\{m_i\}$ is the vector of the masses lumped at the layer interfaces, and $[M]$ is the associated diagonal mass matrix.

The Hadjian method can directly model the fundamental mode shape and can be implemented using a spreadsheet [Dihoru *et al.*, 2016; Nawras *et al.*, 2016; Motazedian *et al.*, 2011]. However, an exponential calculation (Eq. (3)) needs to be repeatedly applied for a multilayer soil profile, which makes the Hadjian method difficult to apply. In the following section, a simpler but still accurate method for estimating the fundamental mode shape of layered soil profiles is described.

3. An Equation for the Natural Frequencies and Mode Shapes of Layered Soil Profiles

In this section, an equation for the estimation of natural frequencies and mode shapes is derived to produce a simple method for the estimation of the fundamental mode shape of layered soil profiles. To derive the equation, a multilayer soil profile on rigid bedrock, which is assumed to vibrate freely in the natural mode, is considered as depicted in Fig. 2.

To establish an equilibrium between the inertial and elastic forces at the ith interface in this model, the inertial force of the soil layers above the ith interface, $F(z_i)$, must be equal to the elastic force, $T(z_i)$, acting on this interface:

$$F(z_i) = T(z_i), \tag{6}$$

where z_i is the depth of the ith interface.

The inertial force of the soil layers above the ith interface can be calculated as

$$F(z_i) = \int_{z_1}^{z_2} -\rho(z)\frac{\partial^2 u}{\partial t^2}\,dz + \int_{z_2}^{z_3} -\rho(z)\frac{\partial^2 u}{\partial t^2}\,dz + \cdots + \int_{z_{i-1}}^{z_i} -\rho(z)\frac{\partial^2 u}{\partial t^2}\,dz, \tag{7}$$

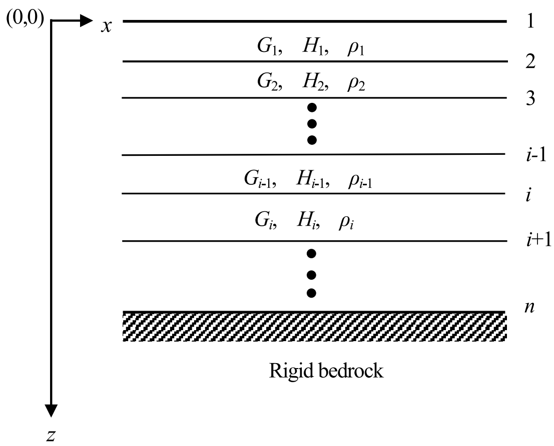

Fig. 2. A multilayer soil profile on a rigid bedrock.

where u is the displacement of the soil layers that can be given by

$$u(z,t) = X(z)\sin(\omega t + \varphi). \tag{8}$$

Here, ω is the natural frequency of the layered soil profile, and X is the corresponding mode shape.

By substituting Eq. (8) into Eq. (7), $F(z_i)$ can be expressed as

$$F(z_i) = \left(\int_{z_1}^{z_2} \rho(z)\omega^2 X(z)dz + \int_{z_2}^{z_3} \rho(z)\omega^2 X(z)dz + \cdots + \int_{z_{i-1}}^{z_i} \rho(z)\omega^2 X(z)dz\right) \times \sin(\omega t + \varphi). \tag{9}$$

For simplicity, the mode shape, X, is assumed to vary linearly with the depth within each soil layer; Eq. (9) can then be simplified as follows:

$$F(z_i) = \frac{1}{2}\omega^2 \sin(\omega t + \varphi)\sum_{j=1}^{i-1} \rho_j (X(z_j) + X(z_{j+1}))H_j. \tag{10}$$

Similarly, the elastic force acting at the ith interface can be simply calculated as

$$T(z_i) = \sin(\omega t + \varphi)\frac{G_i}{H_i}(X(z_i) - X(z_{i+1})), \tag{11}$$

where G_i is the shear modulus of the ith soil layer, $G_i = \rho_i V_i^2$, and V_i is the shear wave velocity.

Substituting Eqs. (10) and (11) into Eq. (6) gives

$$\frac{G_i}{H_i}(X(z_i) - X(z_{i+1})) = \frac{1}{2}\omega^2 \sum_{j=1}^{i-1} \rho_j (X(z_j) + X(z_{j+1}))H_j. \tag{12}$$

From Eq. (12), the following are obtained:

$$X(z_{i+1}) = X(z_i) - \frac{H_i K_i}{G_i}, \tag{13a}$$

$$K_i = \frac{1}{2}\omega^2 \sum_{j=1}^{i-1} \rho_j (X(z_j) + X(z_{j+1})) H_j. \tag{13b}$$

Because both the fundamental frequency, ω, and mode shape, X, are unknown, Eq. (13) has no direct solution. However, ω and X can be obtained using the following steps:

(1) If a preliminary value is assumed for ω, all the values of the fundamental mode shape, $X(z_i)$, can be recursively calculated by setting the value at the surface to be one, i.e. by setting $X(z_1) = 1$.
(2) It is well known that the value of the fundamental mode shape at the base layer, $X(z_n)$, equals zero for natural vibration. Thus, if $X(z_n)$ is zero, the initial assumption of ω in Step (1) represents the accurate natural frequency of the vibration; otherwise, the assumed value of ω is adjusted.
(3) Repeat Steps (1) and (2) until the resulting value of $X(z_n)$ becomes zero. At this point, the natural frequencies and mode shapes can be obtained.

To illustrate how to adjust the assumed value of ω according to the calculated value of $X(z_n)$, we perform a calculation example using Eq. (13) and a sample selected from the strong-motion seismograph networks [K-NET, KIK-net]. The shear wave velocity profile of the sample site is shown in Fig. 3. The relationship between the assumed frequency, ω, and the calculated value of the fundamental mode at the base, $X(z_n)$, is shown in Fig. 4. It is observed that the positive estimated values of $X(z_n)$

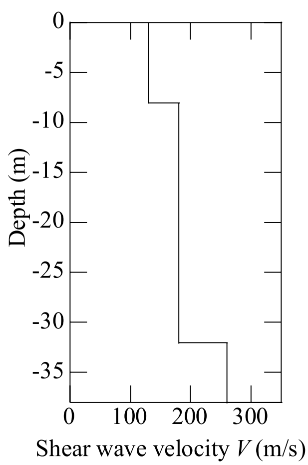

Fig. 3. Shear wave velocity of a soil profile.

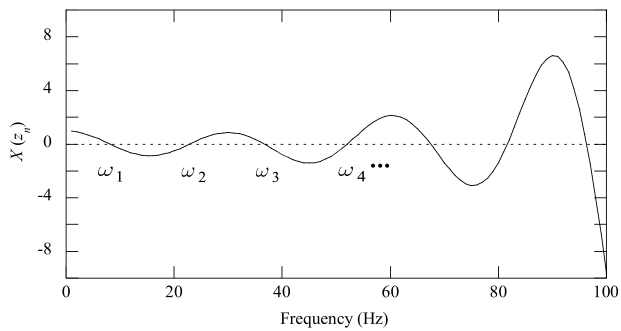

Fig. 4. Estimated value of the fundamental mode shape at the base as a function of frequency.

correspond to values of ω smaller than the true frequency for odd modes, with the converse applies for even modes. Similarly, if the estimated value of $X(z_n)$ is negative, the assumed ω will be larger/smaller than the true frequency for odd/even modes. On the basis of this relation, the natural periods and mode shapes can be gradually determined.

In principle, this method can provide any natural frequency and mode shape with a desired degree of accuracy. Given that the values of the mode shape between two adjacent soil interfaces are assumed to vary linearly in the derivation of Eqs. (10) and (11), the accuracy of the results obtained using Eq. (13) depends only on the soil layer height to be discretized.

4. Method for Obtaining the Fundamental Mode Shape

4.1. *Estimation of the fundamental mode shape*

Although the method developed in Sec. 3 can produce any natural frequency and mode shape with a desired degree of accuracy, the requirement of repeated assumptions and judgments increases the difficulty of applying this method in practical engineering. Note that if a value of the fundamental frequency is given, Eq. (13) can directly produce the fundamental mode shape. In addition, many simple methods have been developed for the estimation of the fundamental period [Dobry *et al.*, 1976]. Thus, using Eq. (13) in conjunction with a simple method for the fundamental period should allow for the simple calculation of the fundamental mode shape. In theory, the more accurate the selected simple method for the fundamental period, the more accurate the obtained fundamental mode shape should be. However, the analysis of a variety of simple methods for estimating the fundamental period reveals that the accuracy of the fundamental mode shape given by Eq. (13) is not sensitive to that of the fundamental period. Therefore, this study utilizes the simplest and most commonly used method to calculate the fundamental period

$$T = \frac{4H^2}{\sum_{i=1}^{n} V_i H_i}. \tag{14}$$

Thus, the fundamental mode shape of layered soil profiles can be estimated as follows:

(1) Estimate the fundamental period using Eq. (14).
(2) Substitute the obtained fundamental period into Eq. (13) to determine the fundamental mode shape.
(3) Given that the fundamental period estimated using Eq. (14) is not precise, any error in the obtained fundamental period will be transferred to the fundamental mode shape. To reduce this error, the obtained values of the fundamental mode shape should be modified. As the value of the fundamental mode shape at the base layer, $X_1(z_n)$, theoretically equals zero for natural vibration, the modification of the fundamental mode shape values will proceed as follows:

$$X_1^m(z_i) = X_1(z_i) - X_1(z_n), \tag{15}$$

where $X_1^m(z_i)$ is the modified value of the fundamental mode shape at the ith soil interface.

The proposed procedure contains three simple equations, i.e. Eqs. (13)–(15), each of which comprises only arithmetic operations, which makes it easy to estimate the result. By comparing with the Hadjian method introduced in Sec. 2 (i.e. Eqs. (1)–(4)), the simplicity of the proposed method can be observed.

After identifying the fundamental mode shape, the corresponding participation factor can be estimated based on Eq. (5) as used by Hadjian [2002]. Herein, by further substituting the mass lumped at the layer interface, $m_i = 0.5(\rho_{i-1}H_{i-1} + \rho_i H_i)$, into Eq. (5), a direct expression for the participation factor corresponding to the first mode, p_L, can be obtained as

$$p_L = \frac{X_1^m(z_1)\rho_1 H_1 + \sum_{i=2}^{n-1}(\rho_{i-1}H_{i-1} + \rho_i H_i)X_1^m(z_i)}{(X_1^m(z_1))^2\rho_1 H_1 + \sum_{i=2}^{n-1}(\rho_{i-1}H_{i-1} + \rho_i H_i)(X_1^m(z_i))^2}. \tag{16}$$

Although Eq. (16) seems complicated at first glance, it also contains only the arithmetic operations.

4.2. *Application of the proposed method*

This subsection presents a calculation example in which the proposed procedure is applied to a multilayer soil profile described in Sec. 3 for the shear wave velocity that is depicted in Fig. 3. The soil data for each layer are listed in Table 1. The calculation steps are detailed below:

- Step 1: The fundamental period is calculated by Eq. (14), resulting in a fundamental period of 0.8347 s.
- Step 2: By substituting the fundamental period into Eq. (13), the values of the fundamental mode shape can be recursively obtained by setting the value at the surface to one, i.e. $X_1(z_1) = 1$. The obtained results are presented in Table 2. To attain any required degree of accuracy, the soil profile can be discretized into any

Table 1. Soil data for a sample soil profile.

Layer no.	Thickness H_m (m)	Shear wave velocity V_m (m/s)	Density ρ_m (KN/m^3)
1	8	130	18.62
2	24	180	18.62
3	6	260	18.62

Table 2. Fundamental mode shape results for a sample soil profile.

Depth (m)	Rayleigh procedure	Results of step 2	Results of step 3	Relative error (%)
0	1.000	1.000	1.000	0.00
1	0.998	1.000	1.000	0.20
2	0.993	0.997	0.996	0.32
3	0.983	0.990	0.989	0.58
4	0.970	0.980	0.978	0.77
5	0.954	0.967	0.963	0.90
6	0.933	0.950	0.944	1.18
7	0.909	0.930	0.922	1.41
8	0.882	0.907	0.896	1.59
9	0.866	0.894	0.881	1.71
10	0.848	0.879	0.864	1.86
11	0.828	0.862	0.845	2.07
12	0.807	0.844	0.825	2.19
13	0.784	0.824	0.803	2.37
14	0.760	0.803	0.779	2.47
15	0.734	0.780	0.754	2.66
16	0.707	0.756	0.727	2.77
17	0.678	0.731	0.698	2.98
18	0.648	0.704	0.668	3.15
19	0.617	0.676	0.637	3.26
20	0.584	0.647	0.605	3.51
21	0.550	0.617	0.571	3.75
22	0.515	0.586	0.536	3.98
23	0.479	0.553	0.499	4.22
24	0.442	0.520	0.462	4.48
25	0.405	0.485	0.423	4.52
26	0.366	0.450	0.384	4.86
27	0.327	0.414	0.343	5.02
28	0.287	0.377	0.302	5.30
29	0.246	0.340	0.260	5.77
30	0.205	0.302	0.218	6.10
31	0.164	0.263	0.174	6.16
32	0.122	0.224	0.130	6.80
33	0.102	0.205	0.109	6.86
34	0.082	0.186	0.088	6.71
35	0.061	0.166	0.066	8.03
36	0.041	0.147	0.044	7.56
37	0.020	0.127	0.022	10.50
38	0.000	0.108	0.000	0.00

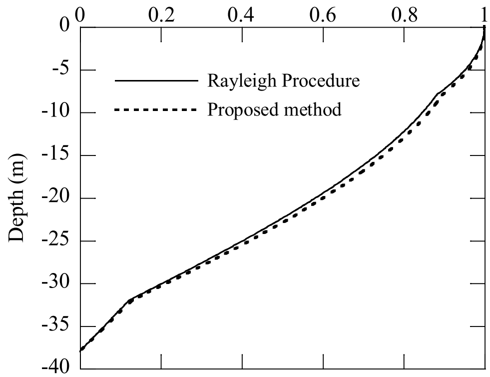

Fig. 5. Comparison of the fundamental mode shapes estimated using the proposed method with Rayleigh procedure results.

number of layers; in this example, the soil profile is discretized into $1-m$ deep layers.

- Step 3: The values of the fundamental mode shape obtained from Step 2 are modified using Eq. (15). The obtained final fundamental mode shapes are presented in Table 2.

The calculations that are performed in each step of the proposed procedure can be easily implemented. To verify the accuracy of the proposed procedure, the fundamental mode shapes were also calculated using the exact Rayleigh procedure and were further compared to estimate the relative error. Table 2 presents the Rayleigh procedure results and relative errors, which indicate that the fundamental mode shapes obtained using the proposed method are remarkably accurate and depict a maximum error of only 10.5%. Figure 5 depicts a comparison of the fundamental mode shapes with the Rayleigh procedure results, which further confirms the close agreement between the two methods. The proposed method is further verified in the next section.

5. Numerical Examples and Discussion

5.1. *Designed soil profile*

To validate the proposed method, the uniform 60.98-m soil profile with a shear wave velocity of 304.8 m/s from Sec. 4.1 of Hadjian [2002] is used. The fundamental mode shape of the soil profile, discretized by a number of equal-height layers, is estimated by the proposed method; the obtained mode shapes together with the correct mode shapes are plotted in Fig. 6. The figure indicates that the results of the fundamental mode shape by the proposed method agree very well with the correct values. In addition, the comparison of the mode shapes having different layer discretizations demonstrates that the accuracy of the proposed method increases with the number of layers, but the variation is not significant. Thus, the proposed

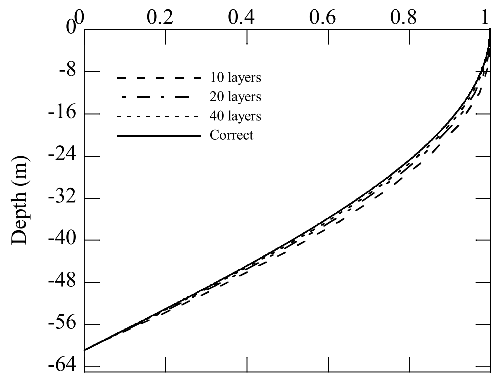

Fig. 6. Comparison of the fundamental mode shapes for several discretizations of a uniform soil profile.

method is stable with respect to the numbers of layers. Moreover, the mode shapes obtained by the proposed method are compared with those obtained by the Hadjian method (Fig. 5 of the [Hadjian, 2002]); no obvious differences are observed between the results that are obtained using both the methods. Further accuracy investigation of the proposed method using actual soil profiles is described in the following section.

5.2. *Actual soil profiles*

Four actual soil profiles, Site-1, Site-2, Site-3, and Site-4 (Table 3), were selected from the strong-motion seismograph networks [K-NET, KIK-net] and used to investigate the accuracy of the proposed method. The fundamental mode shapes were estimated using the proposed method, the Hadjian method [Hadjian, 2002], and the exact Rayleigh procedure [Dobry *et al.*, 1976] by discretizing the soil profiles into several 1-m soil layers. The results are shown in Figs. 7(a) and 7(b). The horizontal axis represents the estimated values of the fundamental mode shapes, and the shape at the ground surface is normalized to one, whereas the vertical coordinate

Table 3. Soil data for the four representative soil profiles.

Site no.	Layer no.	Thickness H_i (m)	Shear wave velocity V_i (m/s)	Density ρ_i (KN/m^3)
Site-1	1	4	98	15.68
	2	2	98	18.62
	3	6	216	18.62
Site-2	1	22	170	18.62
	2	6	250	18.62
	3	22	300	18.62
Site-3	1	6	136	15.68
	2	30	267	18.62
	3	16	292	18.62
Site-4	1	20	110	15.68
	2	170	380	18.62

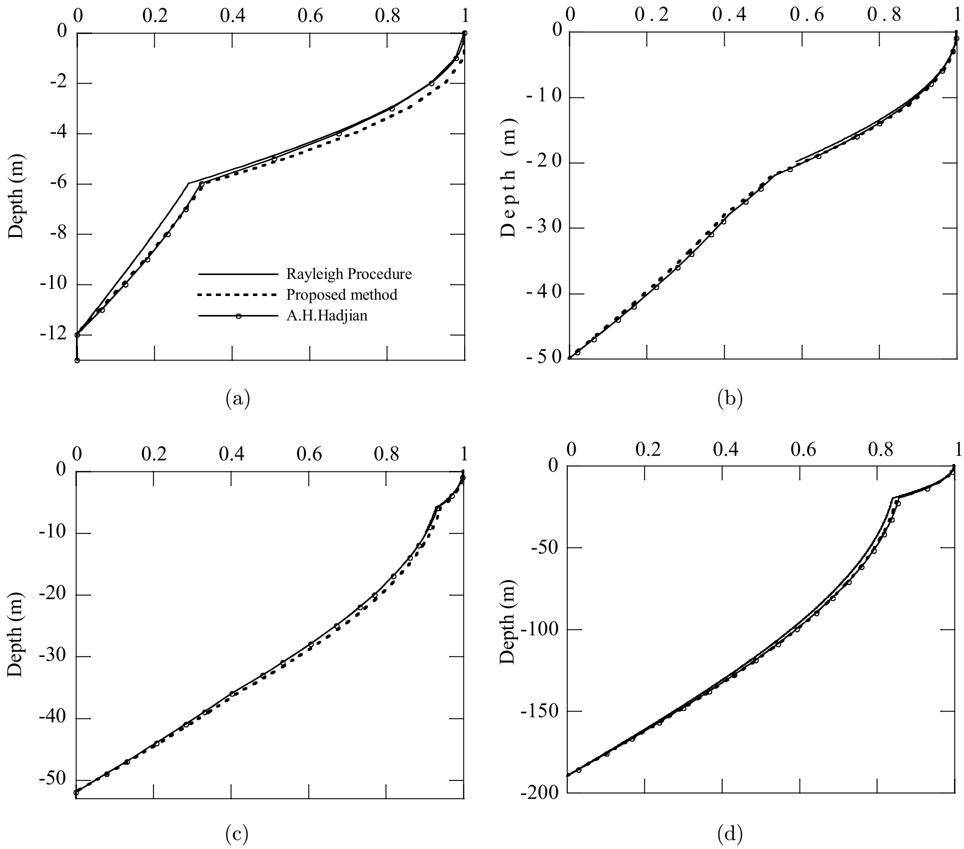

Fig. 7. Fundamental mode shapes of the four representative soil profiles estimated using different methods. (a) Site-1, (b) Site-2, (c) Site-3, and (d) Site-4.

represents the depth. The results produced using the proposed method are observed to be in close agreement with those obtained using the Rayleigh procedure. In addition, comparison of the mode shapes by the proposed and Hadjian methods demonstrates that the accuracies of the results produced by the two methods are nearly identical.

To further compare the fundamental mode shape accuracies of the proposed and Hadjian methods, an additional 63 representative soil profiles were selected from the strong-motion seismograph networks [K-NET, KIK-net]. The shear wave velocity profile of each site is presented in Fig. 8. The unit weights are not given for some sites; instead, these were empirically determined according to Yuki *et al.* [2003] as 15.68 KN/m^3 for clay and 18.62 KN/m^3 for sand. The fundamental periods of the selected soil profiles were calculated using the SHAKE program [Idriss and Sun, 1992], and the results varied widely from 0.05 to 1.72 s.

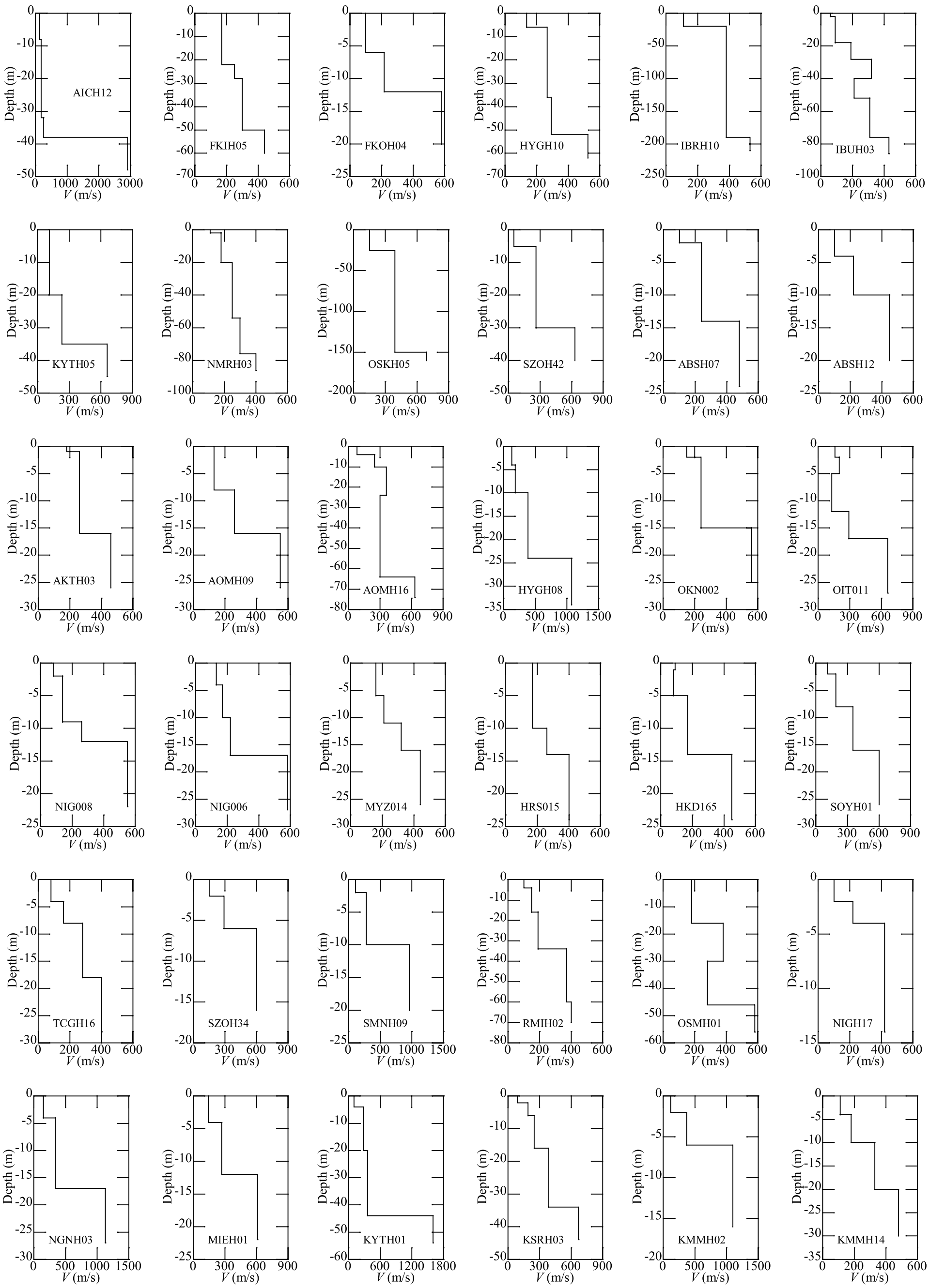

Fig. 8. Shear wave velocity profiles used for analyses.

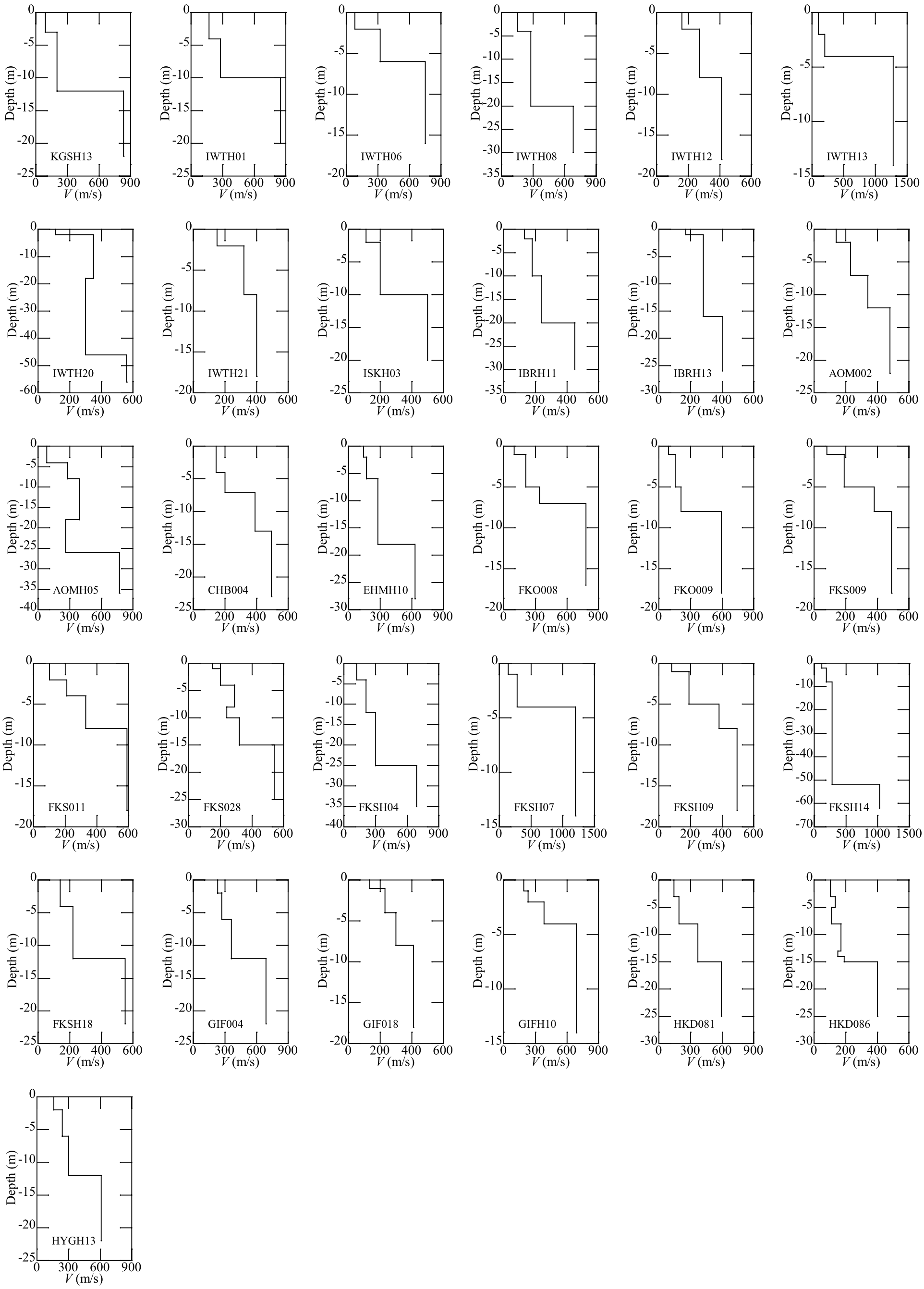

Fig. 8. (*Continued*)

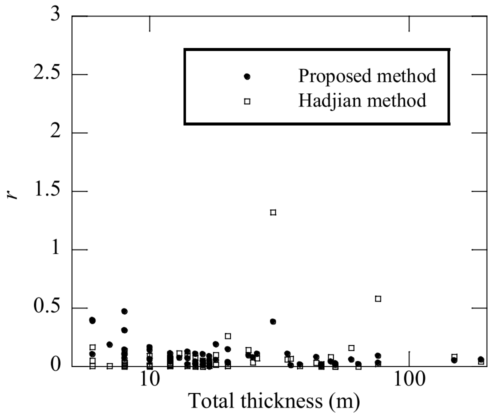

Fig. 9. Values of r for 67 soil profiles.

As a measure of the estimation accuracy of the fundamental mode shape, the parameter r can be defined as follows:

$$r = \sum_{i=1}^{n} \frac{(X_1^{\mathrm{EV}}(z_i) - X_1^{\mathrm{EX}}(z_i))^2}{X_1^{\mathrm{EX}}(z_i)}, \tag{17}$$

where $X_1^{\mathrm{EV}}(z_i)$ and $X_1^{\mathrm{EX}}(z_i)$ represent the evaluated and exact values of the fundamental mode shape at the ith soil interface, respectively. Equation (17) depicts that values of r that are close to zero will correspond to the more accurately calculated fundamental mode shape results.

The values of r obtained by applying the proposed and Hadjian methods to the 67 soil profiles are shown in Fig. 9. In the figure, the horizontal coordinate represents the total thickness of the soil profile, whereas the vertical coordinate represents the

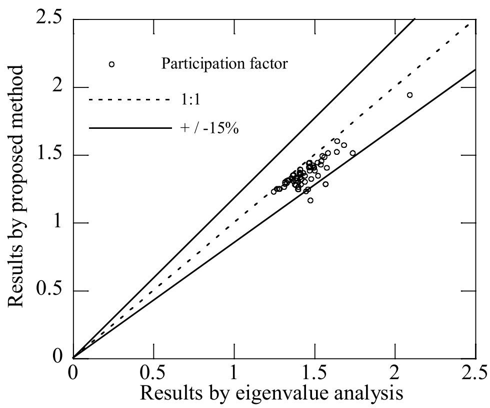

Fig. 10. Comparisons of the participation factors calculated using the proposed method and eigenvalue analysis.

calculated value of r. For several soil profiles with small total thicknesses, the proposed method produces values of r that are slightly higher than those produced using the Hadjian method. However, for several soil profiles with large total thicknesses, the proposed method produces values of r that are lower than those produced by the Hadjian method. Generally, no obvious differences are observed between the values of r calculated using the two methods, which indicates that the accuracies of the two methods are almost identical.

The participation factors corresponding to the first mode were further estimated for the 67 soil profiles using Eq. (16). Figure 10 compares the obtained participation factors with those estimated by performing eigenvalue analysis, which involves discretization of the continuous soil profile into a lumped-parameter MDOF model. It is seen that the modal participation factors obtained using Eq. (16) are remarkably accurate, with 97% of the estimated values within 15% of the results obtained using eigenvalue analysis.

6. Conclusions

In this study, a simple method was developed for calculating the fundamental mode shape of layered soil profiles. On the basis of our analysis and discussion, the following findings and conclusions can be presented:

(1) A new equation for estimating the natural frequencies and mode shapes of layered soil profiles was derived.
(2) By using the derived equation in conjunction with a simple method for the fundamental period, a simple approach was developed to estimate the fundamental mode shape of layered soil profiles. The proposed approach can be conveniently implemented using arithmetic operations.
(3) The accuracy of the proposed approach was investigated using a large sample of representative layered soil profiles. The results produced by the proposed method were observed to agree closely with the actual results.
(4) The proposed method was compared with the Hadjian method; the two methods were observed to exhibit approximately identical accuracy in estimating the fundamental mode shape; however, the implementation of the proposed method is much more convenient as compared to that of the Hadjian method.

Acknowledgment

The study is partially supported by the National Natural Science Foundation of China (Grant No. 51738001). The support is gratefully acknowledged.

References

Dihoru, L., Bhattacharya, S., Moccia, F., Simonelli, A. L., Taylor, C. A. and Mylonakis G. [2016] "Dynamic testing of free field response in stratified granular deposits," *Soil Dyn. Earthq. Eng.* **84**, 157–168.

Dobry, R., Oweis, I. and Urzua, A. [1976] "Simplified procedures for estimating the fundamental period of a soil profile," *Bull. Seismol. Soc. Am.* **66**(4), 1293–1321.

Francesca, D., Sandro, C. and Graziano, L. [2010] "Static equivalent method for the kinematic interaction analysis of single piles," *Soil Dyn. Earthq. Eng.* **30**, 679–690.

Hadjian, A. H. [2002] "Fundamental period and mode shape of layered soil profiles," *Soil Dyn. Earthq. Eng.* **22**, 885–891.

Idriss, I. M. and Sun, J. I. [1992] *SHAKE91: A Computer Program for Conducting Equivalent Linear Seismic Response Analyses of Horizontally Layered Soil Deposits*, User's Guide, University of California, Davis.

Kenji, M., Kohji, K. and Masanori, I. [2001] "Response spectrum method for evaluating nonlinear amplification of surface strata," *J. Struct. Constr. Eng. AIJ* **539**, 57–62 (in Japanese).

Lam, N. T. K., Wilson, J. L. and Chandler, A. M. [2001] "Seismic displacement response spectrum estimated from the analogy soil amplification model," *Eng. Struct.* **23**(11), 1437–1452.

Madera, G. A. [1970] "*Fundamental Period and Amplification of Peak Acceleration in Layered Systems*," Research Report R 70–37, June (MIT Press, Cambridge, MA).

Mistumasa, M., Izuru, O., Masanori I. *et al.* [2003] "Performance-based seismic design code for buildings in Japan," *Eng. Eng. Seismol.* **4**(1), 15–25.

MLIT [2000] *Notification No.1457-2000, Technical Standard for Structural Calculation of Response and Limit Strength of Buildings*, Ministry of Land, Infrastructure and Transport (in Japanese).

Motazedian, D., Banab, K. K., Hunter, J. A., Sivathayalan, S., Crow, H. and Brooks, G. [2011] "Comparison of site periods derived from different evaluation methods," *Bull. Seismol. Soc. Am.* **101**(6), 2942–2954.

Nawras, H., Omar, L., David, C. and Peter, K. W. [2016] "Modelling ground vibrations induced by harmonic loads," *Geotech. Eng.* **169**, 399–409.

Sarma, S. K. [1994] "Analytical solution to the seismic response of visco-elastiv soil layers," *Geotechnique* **44**(2), 265–275.

Strong-motion Seismograph Networks (K-NET, KIK-net), Available at: http://www.kyoshin.bosai.go.jp/kyoshin/;[accessed 17.05.18].

Tsang, H. H., Adrian, M. C. and Lam, N. T. K. [2006] "Simple models for estimating period-shift and damping in soil," *Earthq. Eng. Struct. Dyn.* **35**, 1925–1947.

Vijayendra, K. V., Sitaram, N. and Prasad, S. K. [2015] "An alternative method to estimate fundamental period of layered soil deposit," *Indian Geotech. J.* **45**(2), 192–199.

Yuki, S., Seiji, T., Kazuyoshi, K. *et al.* [2003] "Simplified method to evaluate ground surface amplification assuming the input motions on the engineering bedrock in the revised enforcement order of the building standard law," *J. Struct. Constr. Eng. AIJ* **565**, 73–78 (in Japanese).

Zhao, J. X. [1997] "Modal analysis of soft-soil sites including radiation damping," *Earthq. Eng. Struct. Dyn.* **26**, 93–113.