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A Simple Approach for Estimating the First Resonance Peak of Layered Soil Profiles

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The first resonance peak, Gs_1, represents the amplification ratio of seismic motion when resonance between input motion and the local site occurs. The Gs_1 is important for understanding amplification characteristics of local site, thus it has been adopted for evaluating site effects in the Japanese Seismic Code. Herein, a simple method for estimating the Gs_1 of layered soil profiles is proposed. By replacing a multi-layer soil profile on bedrock with an equivalent one-layer soil profile, the Gs_1 and fundamental period are easily obtained. To realize the one-layer profile, we develop a procedure to replace a two-layer soil profile on bedrock with an equivalent single-layer profile. This procedure is then applied successively to a multi-layer soil profile to obtain an equivalent single-layer soil profile. The validity of the proposed method is demonstrated by evaluating 67 representative sites. The results obtained using the proposed procedure agree well with those produced by the wave propagation method.

Keywords: Site effects; multi-layer soil profile; first resonance peak; wave propagation.

1. Introduction

It has long been recognized that the effects of the local site on ground motion should be considered in the seismic design of structures. In most seismic codes throughout the world, the site effects are generally considered according to several site classes. For example, in Eurocode 8 [EN 1998-1, 2004] and the International Building Code [IBC, 2012], site effects are reflected in terms of site factors or site coefficients for several site classes. In the Chinese Seismic Code [GB 50011, 2010] and the 1993 Japanese Loads Recommendation [AIJ, 1993], the free-field response spectrum is defined to directly correspond to several site classes, and site effects are implicitly considered by the site classification.

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However, in some regions, such as Japan, geological feature is known to vary significantly through the country; the site effects can hardly be described in detail by several classes of sites. In reality, many important site-specific characteristics can be masked by the site classifications. For example, for a site consisting of soft soil on stiffer rock, soil resonance caused by multiple reflections within the soil medium can cause significant amplification of seismic motion with a frequency near the site's fundamental frequency; however, the resonance effect of a specific site is 'averaged' by the site classification and typically cannot be accurately accounted for by a specific site class. Hence, in some codes including the 2000 Japanese Seismic Code [MLIT, 2000] and Mexico's seismic code [Avilés and Pérez-Rocha, 2012], it has been suggested that site effects be evaluated according to specific sites instead of rough site classifications.

For a specific site, site effects can be quantified via one-dimensional site response analysis by approximating soil deposits as homogeneous or sometimes inhomogeneous layered system [Mylonakis *et al.*, 2013]. The one-dimensional site response analysis is always conducted in time-history domain [Youssef *et al.*, 2002; Duhee *et al.*, 2004; Kwok *et al.*, 2007] or in frequency domain. For both the cases, the input seismic motion for site response analysis is required in the form of time history. But, the seismic motion for structural design is typically given in the form of response spectrum. Thus, the response spectrum defined on bedrock must be converged to time history. It has been recognized that two spectrum-compatible time histories may give significantly different site response results. Thus, a large number of time histories and many times of site response analysis may be required to obtain reliable results. This method not only is cumbersome and computationally expensive but also requires expert knowledge and experience to properly select spectrum-compatible time histories. For practical seismic design, a simple method using representative parameters to reflect site effects is desirable.

The first resonance peak, Gs_1, which is defined as the first peak of the transfer function corresponding to the site fundamental period, represents the amplification ratio of seismic motion when the frequency of input motion is consistent with the fundamental frequency of the local site. The definition of Gs_1 indicates that Gs_1 actually represents the effect of site resonance on seismic motion. Because site resonance can significantly affect site response, especially for flexible soils underlain by high-impedance bedrock, the resonance effect deserves special attention in seismic design [Lam *et al.*, 2001; Hing-Ho *et al.*, 2006, 2017]. In addition, Gs_1 is often found to predict a reasonably well value of the maximum response spectral ratio [Dobry, 1991; Rosenblueth and Arciniega, 1992; Dobry *et al.*, 2000]. Therefore, Gs_1 is a suitable factor to characterize site effects, and it has already been adopted in the Japanese Seismic Code and many studies [Kenji *et al.*, 2001; Yasuhiro *et al.*, 2003; Kehji *et al.*, 2004; Wakako *et al.*, 2010; Haizhong *et al.*, 2017a, 2017b].

Many studies have focussed on the assessment of Gs_1. The theoretical Gs_1 of the layered soil profile can be accurately obtained by calculating the transfer function using matrix method proposed by Thomson [1950] and Haskell [1953] or directly by using the program SHAKE [Idriss and Sun, 1992], although this procedure is cumbersome. To avoid the complicated procedure of the direct method, a simple method for practical engineering is included in the Japanese Seismic Code; in this method, Gs_1 is evaluated by approximating a multi-layer soil profile as an equivalent single-layer profile by weighted averaging the soil shear wave velocity and density. However, this method significantly underestimates Gs_1 when the impedance contrast of the soil layers is large [Yasuhiro *et al.*, 2003; Kehji *et al.*, 2004; Wakako *et al.*, 2010; Haizhong *et al.*, 2017a, 2017b]. Although some improvements have been proposed by Kehji *et al.* [2004], the accuracy of this method remains unacceptable for engineering design [Wakako *et al.*, 2010]. Two simple methods for estimation of the Gs_1 have been proposed in our previous works [Haizhong *et al.*, 2017a, 2017b]. Although both the methods give better results of Gs_1 than those by the simple methods introduced earlier, a more accurate simple method for evaluating the Gs_1 is desirable for practical engineering.

In this paper, a more accurate simple procedure for determining the Gs_1 of layered soil profiles is proposed. The rest of the paper is organized as follows. First, in Sec. 2, the current methods for estimating Gs_1 are reviewed. Next, a new, simple method for determining the Gs_1 of layered soil profiles is developed based on the following basic idea: by replacing the complicated multi-layer soil profile on bedrock with an equivalent one-layer soil profile, the Gs_1 along with the fundamental period can be easily obtained. The process employed to realize this one-layer equivalence is detailed in Secs. 3 and 4. In Sec. 3, a procedure to replace a two-layer soil profile on bedrock by an equivalent single-layer soil profile, which is called the two-to-single (TTS) procedure, is derived. Section 4 then describes the procedure by which a multi-layer soil profile on bedrock is replaced by an equivalent single-layer profile by successively applying the TTS procedure. In this procedure, the top two layers are assumed to overlie bedrock and are replaced by an equivalent single layer using the derived TTS procedure. The equivalent single layer and the third layer can be treated as a new two-layer soil, which is also replaced by an equivalent single layer. By applying the TTS procedure successively to the remaining lower layers, the multiple soil layers are finally replaced by an equivalent single layer. The fundamental period and Gs_1 can then be easily obtained. In Sec. 5, to demonstrate the validity of the proposed method, 67 representative soil profiles are evaluated, and the results are shown to agree well with those obtained by the wave propagation method. Finally, the conclusions are presented in Sec. 6.

2. Review of Current Methods for Calculating Gs_1

Many studies have focussed on determining the first resonance peak, Gs_1, of layered soil profiles. For the simplest soil profile (i.e. a single-layer soil profile on bedrock), simple equations for Gs_1 and fundamental period, T_1, are given by:

$$Gs_1 = \frac{1}{1.57\,h + a_G}, \tag{1}$$

$$T_1 = \frac{4H}{V}, \tag{2}$$

where H is the soil thickness, V is the soil shear wave velocity, h is the soil damping ratio, and a_G is the impedance ratio of the soil layer with respect to the bedrock, which is defined as:

$$a_G = \frac{\rho V}{\rho_B V_B}, \tag{3}$$

where V_B and ρ_B are the shear wave velocity and density of the bedrock, respectively.

For a multi-layer soil profile on bedrock, the most widely used method for determining the Gs_1 and T_1 is to replace multiple soil layers with an equivalent single layer by calculating the weighted averages of soil shear wave velocity and density as:

$$V = \frac{\sum_{m=1}^{N} V_m H_m}{\sum_{m=1}^{n} H_m}, \tag{4}$$

$$\rho = \frac{\sum_{m=1}^{N} \rho_m H_m}{\sum_{m=1}^{n} H_m}, \tag{5}$$

where m is the soil layer number, each soil layer has finite thickness H_m, shear wave velocity V_m, and density ρ_m, and N is the number of soil layers. In addition, the damping ratio h is also calculated as the weighted average of all soil layers [Jose *et al.*, 1973; BMHI, 2001] as follows:

$$h = \frac{\sum_{i=1}^{N} h_m E_m}{\sum_{i=1}^{m} E_m}, \tag{6}$$

where E_m is the energy stored in the mth layer [Jose *et al.*, 1973; Mistumasa *et al.*, 2003]. For linear analysis, the soil damping ratio of each layer, h_m, is constant, and is generally considered equal to 0.02. For nonlinear analysis, the damping ratio of each layer is dependent on the shaking level and can be approximately estimated using the equivalent-linear method.

It should be noted that replacing a multi-layer soil profile with an equivalent single-layer soil profile using Eqs. (4) and (5) does not guarantee that the T_1 and Gs_1 of the equivalent single-layer soil profile are equal to those of the original multi-layer soil profile. As mentioned earlier, this method is known to underestimate Gs_1, especially when the impedance contrast of the soil layers is large [Yasuhiro *et al.*, 2003; Kehji *et al.*, 2004; Wakako *et al.*, 2010].

Then, another approximate method for estimating Gs_1 of a multi-layer soil profile on bedrock is proposed by Kehji *et al.* [2004], and the equation is expressed as:

$$Gs_1 = \prod_{m=1}^{N} \frac{1}{1.57h'_m + a_m}, \tag{7}$$

where h'_m is the equivalent damping ratio of the mth soil layer, a_m is the impedance ratio of the mth soil layer with respect to the $(m+1)$th soil layer. Kehji *et al.* [2004] assume that, for a multi-layer soil profile, Gs_1 of each soil layer can be calculated by Eq. (1), and Gs_1 of the total soil profile is equal to the product of that of each soil layer. However, even for the same soil profile, the Gs_1 calculated by this method differs depending on how the soil profile is discretized [Kehji *et al.*, 2004].

Two simple methods for estimating the Gs_1 of layered soil profiles have been proposed in our previous works [Haizhong *et al.*, 2017a, 2017b]. One of the previous methods calculates the Gs_1 by replacing the multiple soil layers with equivalent two layers [Haizhong *et al.*, 2017a]. The interface of the equivalent two layers is located between two adjacent soil layers whose impedance contrast is largest among all soil layers. The other method calculates the Gs_1 by replacing the layered shear wave velocity profile with an equivalent linearly varying profile [Haizhong *et al.*, 2017b]. The equivalent shear wave velocity profile is determined by regressing the values at midpoint of each soil layer. Although both the methods give better estimation of Gs_1 than those by the simple methods introduced earlier, a more accurate method for evaluating the Gs_1 is desirable for practical engineering.

3. Development of the TTS Procedure

To overcome the shortcomings of existing methods introduced earlier, we introduce a method to equate the fundamental period and $Gs_1$of a multi-layer soil profile with those of an equivalent single-layer soil profile; that is, the method replaces a multi-layer soil profile by an equivalent single-layer soil profile with same fundamental period and Gs_1. Therefore, the Gs_1 of the multi-layer soil profile can be simply calculated from that of the equivalent single-layer soil profile.

For this purpose, we first develop a procedure to replace a two-layer soil profile on bedrock with an equivalent single-layer soil profile with the same fundamental period and Gs_1. This method is called two-to-single (TTS) procedure. By successively applying the TTS procedure, a multi-layer soil profile can be replaced by an equivalent single-layer soil profile with approximately the same fundamental period and Gs_1. This section details the theory underlying the TTS procedure.

3.1. *Gs_1 of a two-layer soil profile on bedrock*

Figure 1 schematically shows the procedure developed to replace a two-layer soil profile on bedrock (a) with an equivalent single-layer soil profile (b) with the same fundamental period and Gs_1. To develop this procedure, the fundamental

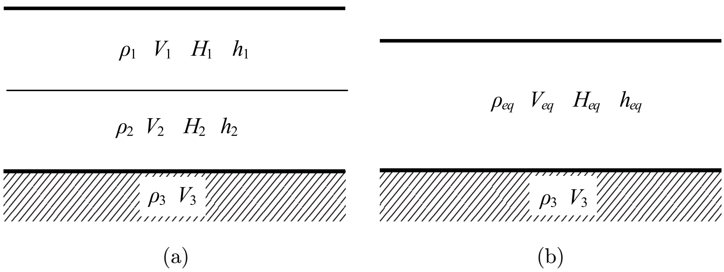

Fig. 1. Illustration of the concept of replacing a two-layer soil profile on bedrock with an equivalent single-layer soil profile.

parameters including shear wave velocity $V_{\rm eq}$, thickness $H_{\rm eq}$, density $\rho_{\rm eq}$, and damping ratio $h_{\rm eq}$ of the equivalent single-layer soil profile should be expressed in terms of those of the two-layer soil profile based on the following two equivalence equations:

$$T_{1-2L} = T_{1-\rm eq}, \tag{8}$$

$$Gs_{1-2L} = Gs_{1-\rm eq}, \tag{9}$$

where T_{1-2L} and Gs_{1-2L} represent the fundamental period and first resonance peak of the two-layer soil profile, respectively; $T_{1-\rm eq}$ and $Gs_{1-\rm eq}$ represent the fundamental period and first resonance peak of the equivalent single-layer soil profile, respectively.

To obtain the equations for the fundamental parameters of the equivalent single-layer soil profile according to Eqs. (8) and (9), the equations for $T_{1-\rm eq}$, $Gs_{1-\rm eq}$, T_{1-2L}, and Gs_{1-2L} expressed in terms of the fundamental parameters of soil profiles must be known. Approximate expressions for $Gs_{1-\rm eq}$ and $T_{1-\rm eq}$ are given by Eqs. (1) and (2), respectively. The expression for T_{1-2L} was derived by Madera [1970], and an approximate expression was subsequently developed by Hadjian [2002]. Thus, the only unknown expression is the one for Gs_{1-2L}, which is derived theoretically in this section.

To derive the expression for Gs_{2L-1}, a two-layer soil profile on bedrock, as shown in Fig. 1(a), is considered. For vertically propagating shear waves, the equilibrium equation can be written as:

$$\rho_m \frac{\partial^2 y_m}{\partial t^2} = G_m \frac{\partial^2 y_m}{\partial x^2}, \tag{10}$$

where

$$G_m = G_{mo}(1 + 2ih_m), \tag{11}$$

m is the layer number ($m = 1, 2, 3$); ρ_m, h_m, G_{m0}, and y_m are the density, damping ratio, shear modulus and displacement of the mth layer, respectively; x is the depth below the surface of each layer; t is time; and i is the complex number ($i^2 = -1$).

For harmonic seismic waves, Eq. (10) can be solved, and the displacement y_m and shear strength τ_m of the mth layer can be, respectively, given by:

$$y_m(x,t) = U_m e^{i\omega(t+x/V_m)} + D_m e^{i\omega(t-x/V_m)}, \tag{12}$$

$$\tau_m(x,t) = i\omega\rho_m V_m (U_m e^{i\omega(t+x/V_m)} - D_m e^{i\omega(t-x/V_m)}), \tag{13}$$

where ω is the angular frequency of the harmonic wave; U_m and D_m are the amplitudes of waves traveling upwards and downwards in the mth layer, respectively; and V_m is the shear wave velocity of the mth layer, which is defined as:

$$V_m = \sqrt{\frac{G_m}{\rho}}. \tag{14}$$

According to the boundary condition that shear stress at the ground surface is equal to 0 (i.e. $\tau_1(0,t) = 0$), the amplitudes of waves traveling upwards and downwards at the ground surface are equal:

$$U_1 = D_1. \tag{15}$$

According to two additional boundary conditions, (1) relative displacement at the interface between two adjacent layers is zero and (2) shear stress at the interface between two adjacent layers is continuous, expressed as:

$$\begin{cases} y_m(H_m,t) = y_{m+1}(0,t) \\ \tau_m(H_m,t) = \tau_{m+1}(0,t) \end{cases}, \tag{16}$$

the amplitudes of waves traveling upwards and downward (U_m and D_m, respectively) in each layer are given by:

$$\begin{cases} U_{m+1} = \dfrac{1}{2}[(1+a_m)U_m e^{i\omega H_m/V_m} + (1-a_m)D_m e^{-i\omega H_m/V_m}] \\ D_{m+1} = \dfrac{1}{2}[(1-a_m)U_m e^{i\omega H_m/V_m} + (1+a_m)D_m e^{-i\omega H_m/V_m}] \end{cases}, \tag{17}$$

where H_m is the thickness of the mth soil layer.

Using Eqs. (15) and (17), the wave amplitude traveling upwards at the bedrock, U_3, can be given by:

$$U_3 = U_1((\cos C_1 \cos C_2 - a_1 \sin C_1 \sin C_2) + i(a_1 a_2 \sin C_1 \cos C_2 + a_2 \cos C_1 \sin C_2)), \tag{18}$$

where

$$C_m = \frac{\pi T_m}{2T\sqrt{1+2ih_m}}, \quad T_m = \frac{4H_m}{V_m}, \quad T = \frac{2\pi}{\omega},$$

and a_m is the impedance ratio between adjacent layers, which is defined as:

$$a_m = \frac{\rho_m V_m}{\rho_{m+1} V_{m+1}}, \tag{19}$$

Then, the transfer function for the seismic motions at outcrop bedrock can be obtained as:

$$H_2(\omega) = \frac{U_1 + D_1}{2 \times U_3} = \frac{1}{\begin{array}{l}(\cos C_1 \cos C_2 - a_1 \sin C_1 \sin C_2) \\ \quad + i(a_1 a_2 \sin C_1 \cos C_2 + a_2 \cos C_1 \sin C_2)\end{array}}. \tag{20}$$

Using Eq. (20), the first peak of the transfer function corresponding to the fundamental period can be obtained by making the period T equal to the fundamental period of the two-layer soil profile, T_{1-2L}. For two undamped soil layers ($h_m = 0$) on rigid bedrock ($V_B = \infty$), the equation for T_{1-2L} has been derived by Madera [1970] and is given by:

$$\tan \frac{\pi T_1}{2T_{1-2L}} \tan \frac{\pi T_2}{2T_{1-2L}} = \frac{\rho_2 H_2 T_1}{\rho_1 H_1 T_2}. \tag{21}$$

As the effect of bedrock rigidity on site fundamental period is considered negligible [Sarma, 1994; Vijayendra *et al.*, 2015], Eq. (21) is also available to calculate the fundamental period for the two-layer soil profile on elastic bedrock. Soil damping is first disregarded. Substituting Eq. (21) into Eq. (20), the real part of the denominator in Eq. (20) becomes zero, and the undamped first resonance peak ($h_m = 0$) of the two-layer soil profile, Gs_{1-2L}, can be given by:

$$Gs_{1-2L} = \frac{1}{\left| a_1 a_2 \sin \frac{\pi T_1}{2T_{1-2L}} \cos \frac{\pi T_2}{2T_{1-2L}} + a_2 \cos \frac{\pi T_1}{2T_{1-2L}} \sin \frac{\pi T_2}{2T_{1-2L}} \right|}. \tag{22}$$

Equation (22) is derived by disregarding the soil damping. However, soil damping has been shown to significantly affect the site amplification; thus, the soil damping ratio should be parameterized in the equation for Gs_{1-2L}. For a single-layer soil profile on bedrock (Fig. 1(b)), the soil damping ratio is considered approximately by the term $1.57h$ in Eq. (1). Based on this consideration, the equation for Gs_{1-2L} considering soil damping for the two-layer soil profile is approximated as:

$$Gs_{2L-1} = \frac{1}{\left| a_1 a_2 \sin \frac{\pi T_1}{2T_{2L-1}} \cos \frac{\pi T_2}{2T_{2L-1}} + a_2 \cos \frac{\pi T_1}{2T_{2L-1}} \sin \frac{\pi T_2}{2T_{2L-1}} \right| + 1.57 h_{\text{eq}}}, \tag{23}$$

where h_{eq} is the equivalent damping ratio calculated by Eq. (6).

It should be noted that Eq. (23) is consistent with Eq. (1) when $V_1 = V_2$, $\rho_1 = \rho_2$, and $h_1 = h_2$.

3.2. *Shear wave velocity and thickness for the equivalent single-layer soil profile*

As mentioned above, developing the TTS procedure actually means obtaining expressions for the four parameters (V_{eq}, H_{eq}, ρ_{eq}, and h_{eq}) of the equivalent single-layer soil based on Eqs. (8) and (9). However, it is theoretically impossible to solve two equations containing four unknown parameters. For this reason, additional two equivalence equations are introduced: (1) as soil destiny generally does not exhibit large variations, the destiny of the equivalent single-layer soil, ρ_{eq}, is considered approximately equal to the weighted-average density calculated by Eq. (5); (2) the damping ratio of the equivalent single-layer soil, h_{eq}, is considered approximately equal to the weighted-average damping ratio calculated by Eq. (6). Thus, the remaining two parameters (shear wave velocity V_{eq} and thickness H_{eq}) of the equivalent single layer can be determined using Eqs. (8) and (9).

Substituting Eqs. (1) and (23) into Eq. (9), the shear wave velocity V_{eq} of the equivalent single layer can be obtained by:

$$V_{\mathrm{eq}} = \left| \frac{V_1\rho_1}{\rho_{\mathrm{eq}}} \sin\frac{\pi T_1}{2T_{1-2L}} \cos\frac{\pi T_2}{2T_{1-2L}} + \frac{V_2\rho_2}{\rho_{\mathrm{eq}}} \cos\frac{\pi T_1}{2T_{1-2L}} \sin\frac{\pi T_2}{2T_{1-2L}} \right|. \tag{24}$$

Next, according to Eq. (8) (i.e. $T_{1-2L} = 4H_{\mathrm{eq}}/V_{\mathrm{eq}}$), the thickness H_{eq} of the equivalent single-layer soil can be given by:

$$H_{\mathrm{eq}} = \frac{T_{1-2L}V_{\mathrm{eq}}}{4}. \tag{25}$$

As introduced above, the fundamental period T_{1-2L} in Eqs. (24) and (25) can be obtained using the charts given by Madera [1970] or using the following approximated equations by Hadjian [2002]:

$$\frac{T_{1-2L}}{T_1} = \sqrt{\frac{\pi^2}{8}\left[0.75 + \left(\frac{T_2}{T_1}\right)^2\left(1 + 2\frac{H_1\rho_1}{H_2\rho_2}\right)\right]}, \quad \text{for } H_1/H_2 > 1, \tag{26}$$

$$\frac{T_{1-2L}}{T_1} = \left[1 + \beta\left(\frac{T_2}{T_1}\right)^n\left(1 + \frac{H_1\rho_1}{H_2\rho_2}\right)^n\right]^{\frac{1}{n}}, \quad \text{for } H_1/H_2 \le 1, \tag{27}$$

where

$$n = 4 - 1.8\frac{H_1\rho_1}{H_2\rho_2} \quad \text{and} \quad \beta = 1 - 0.2\left(\frac{H_1\rho_1}{H_2\rho_2}\right)^2.$$

Hence, using Eqs. (5), (6), (24), and (25), an equivalent single-layer soil profile that has the same fundamental period and first resonance peak as the two-layer soil profile can be obtained. The validity of this method is discussed in Sec. 3.3 below.

3.3. *Validity of the proposed method*

To investigate the validity of the developed TTS procedure, a series of two-layer soil profiles on bedrock are considered. It is known from Eq. (20) that, parameters including impedance ratio a_m, damping ratio h_m, and fundamental period T_m affect the site amplification. Here, damping ratios of the two soil layers are considered equal, and the one of bedrock is considered equal to 0. Actually, only four main parameters including impedance ratios a_1, a_2, soil damping ratio h and fundamental period ratio T_2/T_1 affect the results. A wide range of values for the four parameters are considered as shown in Figs. 2 and 3. Then, all considered two-layer soil profiles are replaced with equivalent one-layer soil profiles using the developed TTS procedure, and first resonance peaks are estimated by Eq. (1). The results by the TTS procedure are compared with those obtained by wave propagation theory (Eq. (20)) in Figs. 2

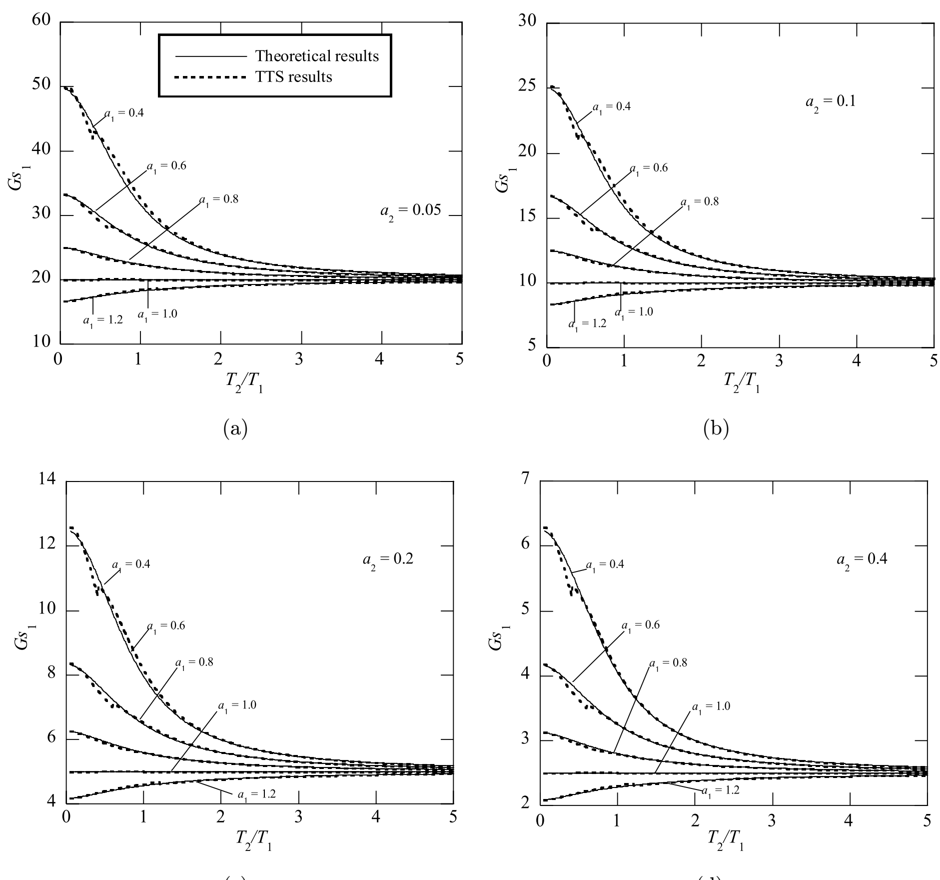

Fig. 2. Comparison of undamped Gs_1 calculated using the developed TTS procedure and wave propagation theory: (a) impedance ratio $a_2 = 0.05$, (b) $a_2 = 0.1$, (c) $a_2 = 0.2$, and (d) $a_2 = 0.4$.

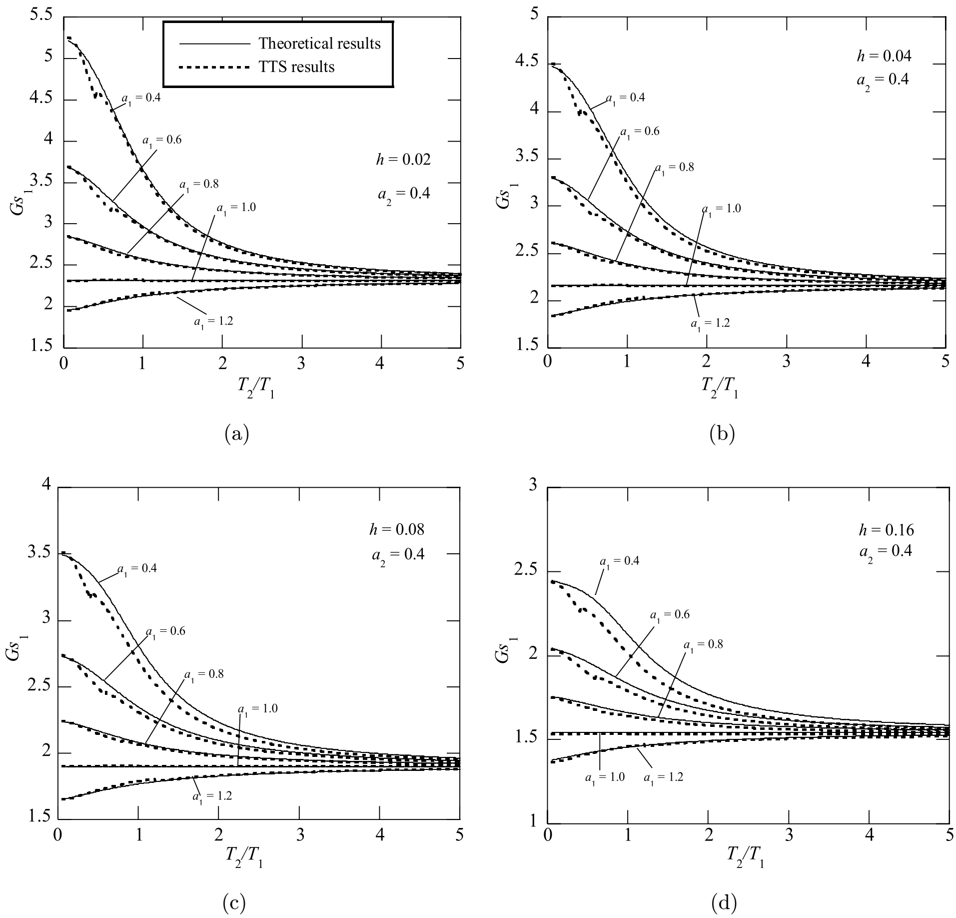

Fig. 3. Comparison of damped Gs_1 calculated using the developed TTS procedure and wave propagation theory: (a) damping ratio $h = 0.02$, (b) $h = 0.04$, (c) $h = 0.08$, and (d) $h = 0.16$.

and 3. Here, as soil nonlinear behavior is not considered in the calculation, the first resonance peak, Gs_1, is not affected by input ground motion; theoretical values of the Gs_1 are obtained directly using the transfer function of two-layer soil profiles expressed by Eq. (20). Figure 2 shows the results obtained by disregarding the soil damping ratio, and Fig. 3 shows those obtained considering a wide range of soil damping ratios. In Figs. 2 and 3, the results obtained by wave propagation theory, referred to as theoretical results, are represented by thin solid lines, and the TTS results are shown by thick dotted lines.

Figure 2 indicates good agreement between the TTS and theoretical results. The maximum relative error in the analyzed soil profiles is approximately 1%. Figure 3 indicates that the error in the TTS results increases as soil damping ratio increases; however, the maximum relative error is approximately 5% when the damping ratio is as much as 16%. Thus, the accuracy of the TTS procedure is considered excellent for engineering use.

4. Gs_1 of Multi-Layer Soil Profiles on Bedrock

4.1. *Method for calculating Gs_1*

This section presents a simple procedure for determining the Gs_1 of multi-layer soil profiles on bedrock by successively applying the TTS procedure developed in Sec. 3. Specifically, for a multi-layer soil on bedrock (Fig. 4(a)), the top two layers are assumed to overlie bedrock and are replaced by an equivalent single layer using the TTS procedure. Subsequently, the equivalent single layer and the third layer can be treated as a new top two-layer soil and can also be replaced by an equivalent single layer. By applying the TTS procedure successively to the remaining lower layers of the soil profile, the multiple soil layers can finally be replaced by an equivalent single layer, and the fundamental period and Gs_1 of the total soil profile can be obtained. The concept of this procedure is illustrated in Fig. 4 and involves the following steps:

(a) For a multi-layer soil on bedrock (Fig. 4(a)), the top two soil layers are assumed to overlie bedrock and can be replaced with an equivalent soil layer using the TTS procedure (i.e. Eqs. (5), (6), (24), and (25)). Next, a new multi-layer soil (Fig. 4(b)) is formed.
(b) For the new multi-layer soil shown in Fig. 4(b), the top two layers are again assumed to overlie bedrock and are replaced by another equivalent single layer using the TTS procedure. Another new multi-layer soil (Fig. 4(c)) is then formed.
(c) By successively applying the TTS procedure until the last soil layer is considered, a final equivalent single-layer soil is obtained, as shown in Fig. 4(d).
(d) Finally, the Gs_1 and fundamental period for the final single-layer soil can be readily obtained using Eqs. (1) and (2), respectively.

The proposed procedure seems more complicated than the current methods introduced in Sec. 2 at first glance. In reality, comparing with the simplest weighted-average method, the proposed method just replaces Eq. (4) by Eq. (24), adds a

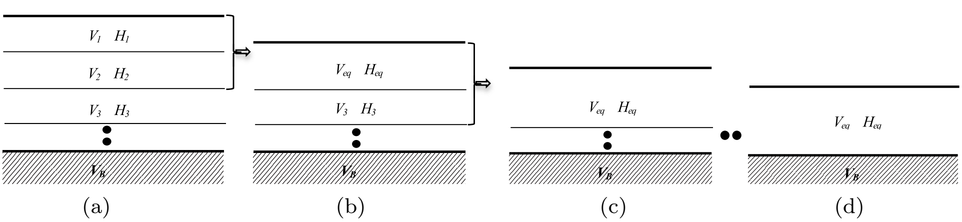

Fig. 4. Illustration of the concept of replacing a multi-layer soil profile on bedrock with an equivalent single-layer soil profile.

simple Eq. (25), and uses these equations more times. As these equations can be easily implemented in a spreadsheet, the proposed method can be simply used in practical engineering.

In addition, it should be noted that the developed procedure for Gs_1 is applicable for not only linear analysis but also the equivalent-linear analysis considering soil nonlinearity. For the equivalent-linear analysis, the proposed procedure is applied just using the final strain-compatible shear modulus and damping ratios after the iteration. Many simple equivalent-linear methods have been developed for estimation of soil nonlinearity (i.e. strain-compatible shear modulus and damping ratio) using bedrock response spectrum directly [Kenji *et al.*, 2001; Wakako *et al.*, 2010]. The method by Kenji *et al.* [2001] has been introduced in the Japanese Seismic Code. Here, any one of these simple methods can be used to consider soil nonlinear in estimation of Gs_1.

4.2. *Application of the proposed method*

This section presents an example calculation in which the proposed procedure is applied to a multi-layer soil profile selected from the Strong-motion Seismograph Networks (K-NET, KIK-net) of Japan. The shear wave velocity of this soil profile is shown in Fig. 5, and the soil data for each layer are listed in Table 1. For simplicity, nonlinear behavior is not considered here, and the damping ratios of all layers are set to 2%. Each step of the calculation is detailed below, and the results of each step are given in Table 2.

Step 1: Assuming that the top two soil layers overlie bedrock and since the thickness of the first layer (2 m) is smaller than that of the second layer (3 m), the fundamental period T_{1-2L} can be calculated using Eq. (27) as $T_{1-2L} = 0.102\,\text{s}$. Using Eqs. (5), (24), and (25), the destiny, shear wave velocity, and

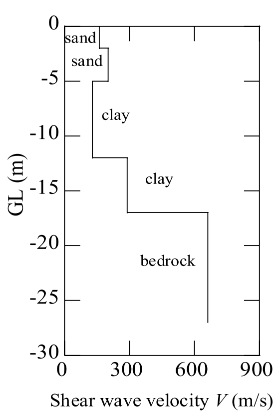

Fig. 5. Shear wave velocity of the example soil profile.

Table 1. Soil data for the example soil profile.

Layer no.	H_m (m)	V_m (m/s)	ρ_m (tf/m^3)	h_m
1	2	160	1.82	2%
2	3	200	1.66	2%
3	7	130	1.66	2%
4	5	290	1.76	2%
5	10	660	2.40	2%

Table 2. Results of the example soil profile at each step by the proposed procedure.

Step	T_{1-2L} (s)	ρ_{eq} (tf/m^3)	V_{eq} (m/s)	H_{eq} (m)	G_{S1}
1.	0.102	1.724	181.2	4.616	—
2.	0.350	1.685	141.0	12.34	—
3.	0.385	1.707	144.2	13.88	—
4.	0.385	—	—	—	5.353

thickness of the equivalent single layer can then be obtained as $\rho_{\mathrm{eq}} = 1.724\,\mathrm{tf/m^3}$, $V_{\mathrm{eq}} = 181.2\,\mathrm{m/s}$, and $H_{\mathrm{eq}} = 4.616\,\mathrm{m}$, respectively.

Step 2: The top two layers are replaced by the new layer obtained in Step 1, and the new layer and the third layer are considered as a new two-layer soil profile. As $H_1 < H_2$ ($4.616\,\mathrm{m} < 7\,\mathrm{m}$), T_{1-2L} can again be calculated using Eq. (27) as 0.350 s. The destiny, shear wave velocity, and thickness of the new equivalent single layer can then be obtained using Eqs. (5), (24), and (25) as $\rho_{\mathrm{eq}} = 1.685\,\mathrm{tf/m^3}$, $V_{\mathrm{eq}} = 141.0\,\mathrm{m/s}$, and $H_{\mathrm{eq}} = 12.34\,\mathrm{m}$, respectively.

Step 3: The new top two layers are replaced by the single layer obtained in Step 2, and the new layer and the fourth layer are considered as a new two-layer soil profile. This time, as $H_1 > H_2$ ($12.34\,\mathrm{m} > 5\,\mathrm{m}$), T_{1-2L} can be calculated using Eq. (26) as $T_{1-2L} = 0.385\,\mathrm{s}$. The destiny, shear wave velocity, and thickness of the final single layer can then be calculated using Eqs. (5), (24), and (25) as $\rho_{\mathrm{eq}} = 1.707\,\mathrm{tf/m^3}$, $V_{\mathrm{eq}} = 144.2\,\mathrm{m/s}$, and $H_{\mathrm{eq}} = 13.88\,\mathrm{m}$, respectively.

Step 4: As nonlinear soil behavior is not considered in this calculation, the equivalent damping ratio is considered to be equal to 2%. Finally, the Gs_1 of the multi-layer soil profile can be calculated from the Gs_1 of the final equivalent single layer obtained in Step 3 using Eq. (1). For this example, $Gs_1 = 5.353$.

Using the proposed procedure, the calculations in each step can be easily implemented using spreadsheet software. To verify the accuracy of the Gs_1 obtained above by the new procedure, the transfer function of the example soil profile is calculated by the SHAKE program, and the resulting Gs_1 and fundamental period are listed in Table 3. The results obtained using the proposed method show good agreement with

Table 3. Comparison of results obtained using different methods.

Method	Estimated fundamental period (s)	Estimated G_{S1}
1. Code method	0.352	4.176
2. Proposed method	0.385	5.353
3. SHAKE	0.394	5.354

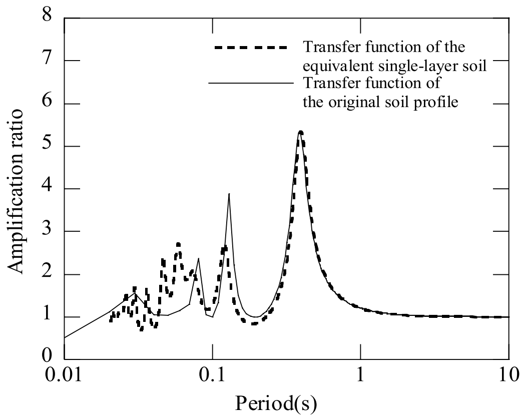

Fig. 6. Comparison of the transfer function of the equivalent single-layer soil profile with that of the original soil profile.

those obtained using the SHAKE program. The results produced by the method in the Japanese Seismic Code are also listed in Table 3. Compared to the SHAKE program, the Japanese Seismic Code method underestimates the Gs_1. In addition, the transfer function of the equivalent single-layer soil profile generated using the proposed procedure is compared with that of the original soil profile in Fig. 6. Figure 6 also indicates good agreement in the first resonance peak. The proposed method is further verified in the next section.

5. Numerical Examples Using the Proposed Procedure

In order to investigate the accuracy of the proposed method, 67 representative soil profiles selected from Strong-motion Seismograph Networks (K-NET, KIK-net) are used. According to JARA [2012], these soil profiles are divided into three site classes, and the shear wave velocity profiles above the engineering bedrock of each site classification are presented in Fig. 7. According to Japanese Seismic Code, engineering bedrock is defined as the layer where the shear wave velocity is greater than approximately 400 m/s [BMHI, 2007]. The unit weights are not given for some sites; these weights are empirically determined according to Yuki *et al.* [2003] as $15.68\,\text{kN/m}^3$ for clay, $18.62\,\text{kN/m}^3$ for sand, $19.60\,\text{kN/m}^3$ for engineering bedrock

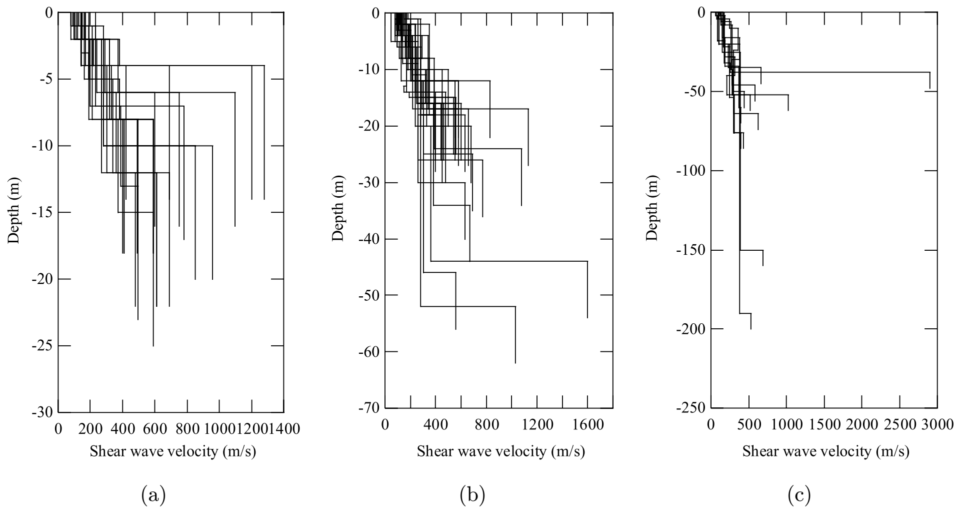

Fig. 7. Shear wave velocity profiles above engineering bedrock used for analyses: (a) first site class, (b) second site class, and (c) third site class.

with shear wave velocity in the range of 400–800 m/s and 21.56 kN/m^3 for engineering bedrock with shear wave velocity greater than 800 m/s. The initial fundamental periods of the selected soil profiles are calculated by the SHAKE program, and the results vary widely from 0.05 s to 1.72 s. Both linear and equivalent-linear analyses are conducted for the accuracy investigation. For linear analysis, damping ratios of all soil layers are simply considered to be 2%. For the equivalent-linear analysis, the simple method by Wakako *et al.* [2010] is adopted to estimate the strain-compatible soil damping ratios and shear modulus. Here, the modulus reduction and damping curves in Japanese Seismic Code are used for the analysis. Both the Level 1 and Level 2 response spectra defined on bedrock in Japanese Seismic Code are used as input motions. For the SHAKE analysis, 10 spectrum-compatible time histories are generated for each of the two load levels. The durations of the Level 1 and Level 2 motions are set to be 60 s and 120 s, respectively. Peak ground accelerations of the ground motions generated using the Level 2 response spectrum vary from 0.34 g to 0.4 g.

The fundamental periods and Gs_1 of the 67 soil profiles are estimated by the proposed procedure and compared with those obtained using the SHAKE program. Figures 8 and 9, respectively, show the linear and equivalent-linear results. The Gs_1 obtained by the proposed method are remarkably accurate. For the linear analysis, the average error is only 4.6%, and 94% of estimated values are within 15% of the SHAKE results. For equivalent-linear analysis, the average errors corresponding to Level 1 and Level 2 motions are, respectively, 4.0% and 3.7%; and for both the levels, 97% of the estimates are within 15% of the SHAKE results. The accuracy in fundamental period is also remarkably good. For the

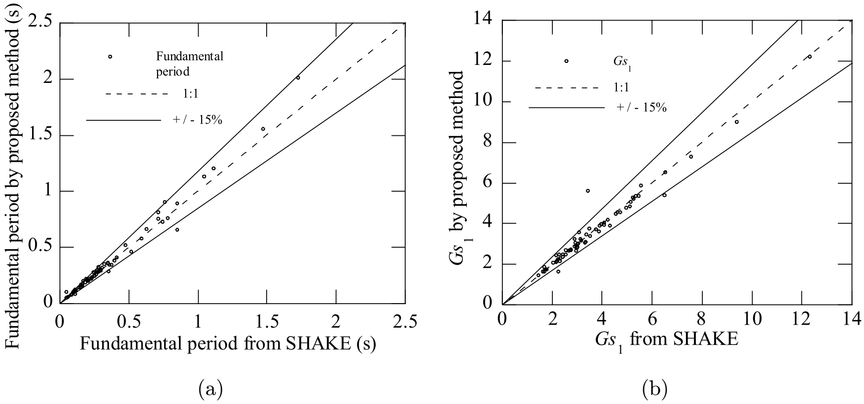

Fig. 8. Comparisons of fundamental period and Gs_1 calculated using the proposed method and SHAKE program for linear analysis.

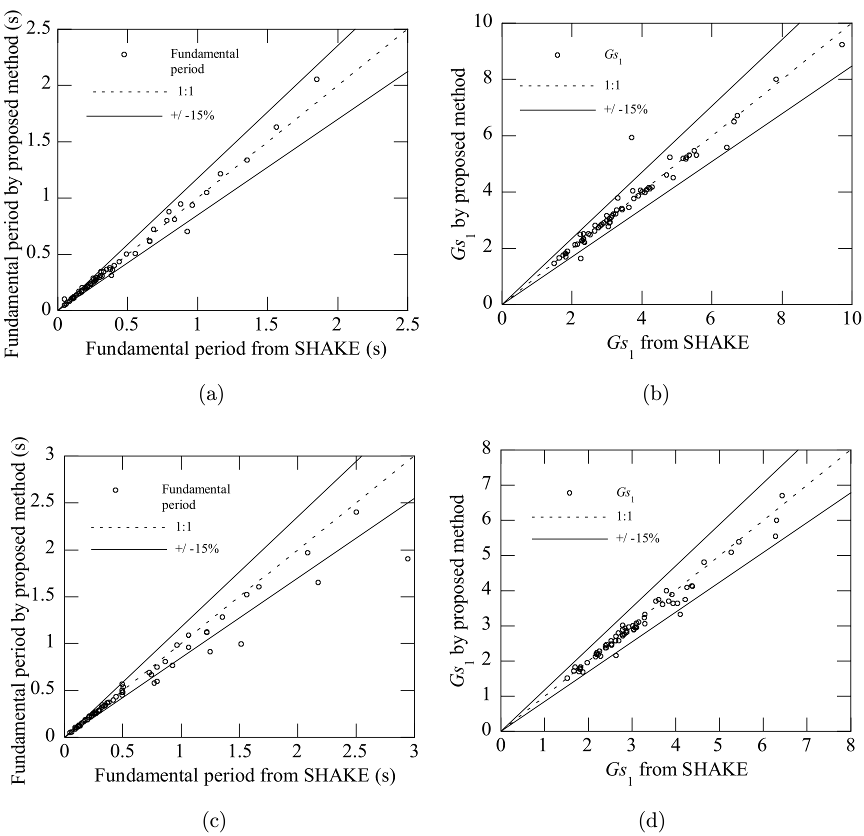

Fig. 9. Comparisons of fundamental period and Gs_1 calculated using the proposed method and SHAKE program for equivalent-linear analysis. (a) Fundamental periods corresponding to the Level-1 motions, (b) First resonance peaks corresponding to the Level-1 motions, (c) Fundamental periods corresponding to the Level-2 motions, (d) First resonance peaks corresponding to the Level-2 motions.

linear analysis, 85% of the estimates are within 15% of SHAKE results. For the equivalent-linear analysis, 94% of the estimates corresponding to Level 1 and 88% of the estimates corresponding to Level 2 are within 15% of SHAKE results. The accuracy of the proposed method is considered sufficient for engineering calculation.

In addition, the fundamental periods and Gs_1 are also estimated using the method in the Japanese Seismic Code and compared with those obtained using the proposed method and the SHAKE program. Figures 10 and 11, respectively, show the linear and equivalent-linear results. The errors in Gs_1 obtained by the code method are significant. For linear analysis, the average error is as large as 17.2%. For equivalent-linear analysis, the average errors corresponding to Level 1 and Level 2 motions are, respectively, 25% and 24%, which are much greater than those for the proposed method. For both the linear and equivalent-linear analyses, most of the Gs_1 estimated by the code method are underestimated by over 15% compared with the SHAKE results, which is consistent with previous studies [Yasuhiro *et al.*, 2003; Kehji *et al.*, 2004; Wakako *et al.*, 2010; Haizhong *et al.*, 2017a]. The errors in the fundamental period obtained by the code method are also significant. For linear analysis, 37% of the estimates have errors greater than 15%. For equivalent-linear analysis, 73% of the Level 1 estimates and 67% of the Level 2 estimates have errors greater than 15%. Generally speaking, the proposed procedure produces accurate estimates of both fundamental period and Gs_1 and is much more accurate than the method used in the Japanese Seismic Code.

The results of Gs_1 by the proposed method shown in Fig. 8(b) are also compared with those by our methods developed previously shown in Fig. 6(a) of both the earlier two papers [Haizhong *et al.*, 2017a, 2017b]. It is found that the results obtained

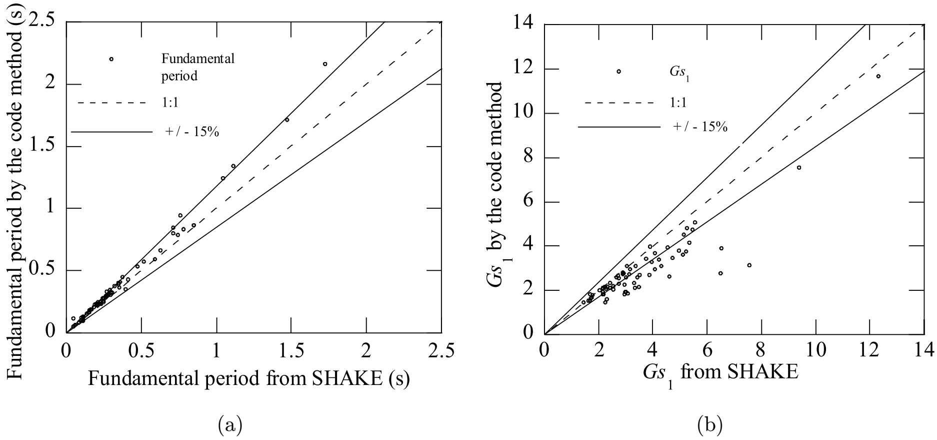

Fig. 10. Comparisons of fundamental period and Gs_1 calculated by the code method and SHAKE program for linear analysis.

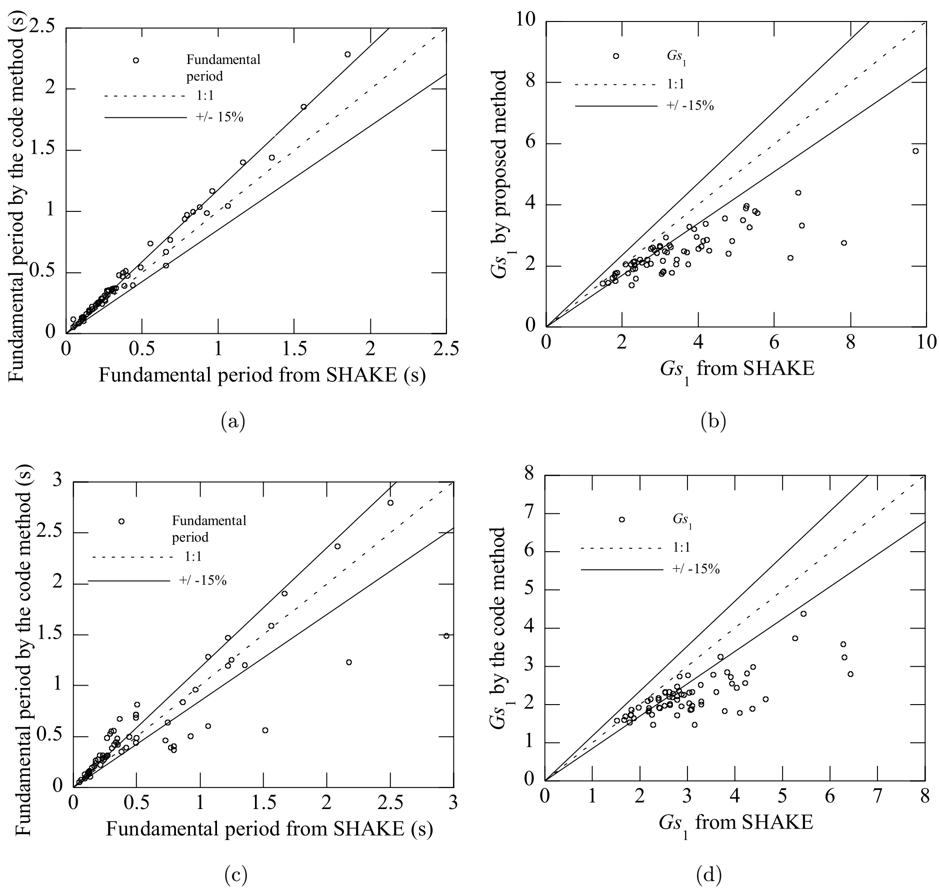

Fig. 11. Comparisons of fundamental period and Gs_1 calculated by the code method and SHAKE program for equivalent-linear analysis. (a) Fundamental periods corresponding to the Level-1 motions, (b) First resonance peaks corresponding to the Level-1 motions, (c) Fundamental periods corresponding to the Level-2 motions, (d) First resonance peaks corresponding to the Level-2 motions.

by the method proposed in this paper are more accurate than those by the previous methods.

It should be noted that the equivalent linear method (SHAKE method) used for calibration above is an approximate method. The method is generally applicable for the cases when the computed shear strain is less than about 1% [Kenji *et al.*, 2001]. In this section, the computed maximum shear strains of most soil profiles using even the Level 2 motions are less than 1%, thus the findings above are valid. However, when the computed shear strains are larger, errors by the equivalent linear method may be significant [Kim and Hashash, 2013], and hence the equivalent linear method may not be appropriate for calibration. Validity of the proposed method for larger ground motions than those considered in this paper needs be investigated in the further study.

6. Conclusion

The content of this paper and the main conclusions are summarized as follows:

(a) A procedure to replace a two-layer soil profile on bedrock with an equivalent single-layer soil profile with the same Gs_1 and fundamental period is developed. The accuracy of the developed procedure is verified using a series of two-layer soil profiles on bedrock.
(b) Based on the developed TTS procedure, a simple procedure for estimating the Gs_1 of a multi-layer soil profile is proposed. The proposed procedure is applied in an example calculation. It is found that the procedure can be easily implemented in a spreadsheet, and the estimated results are highly accurate.
(c) To investigate the validity of the proposed method, the Gs_1 and fundamental periods of 67 representative soil profiles are estimated. The proposed method shows remarkably good accuracy in estimating both the Gs_1 and fundamental period and is clearly more accurate than the current code method.

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